Markov Chain Monte Carlo Analysis of Microstrip Line

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ABSTRACT

Recent advances in the design of microwave solid-state integrated circuits have attracted well-deserved research attention especially the microstrip lines technology. The wide applicable areas of microstrip line have made a modest demand for a more holistic and novel approach in its analysis. In this work, a high grade microstrip line consisting of a track of alumina as an insulating substrate is analysed with the deployment of Markov chain Monte Carlo. Also, we obtained the values of capacitances with and without dielectric substrate, characteristic impedance, and potential distribution along the air-dielectric interface as well as that along the line of symmetry. The results compared with those obtainable from analytical and finite element methods using the same parameters are found to be close.

Keywords: *Shielded microstrip lines, capacitance per unit length, Markov chain, characteristic impedance, Computation, Modelling and simulation.*

1. INTRODUCTION

The spiralling applications of microwave solid-state integrated circuits in recent years have attracted well-deserved research attention especially the microstrip lines technology. The microstrip lines technology has received considerable endorsement as suitable transmission lines for the design of microwave integrated circuits for high frequency applications [1]. The microstrip lines minimize "cross-talk" which is considered a serious concern at high frequency and high component density. This unique advantage that they possess has further strengthened the case in their favor for a more holistic and novel approach in their analysis. They find application areas in circuit components such as filters, couplers, resonators and antennas and other sensitive communication systems such as high-speed digital applications, power radars and satellite communications. Critical to the microstrip technology are capacitances and characteristic impedance. The growing complexity of designs and shapes are lending intricacies into the analytical methods for calculating capacitances and characteristic impedance of shielded microstrip transmission lines. Hence, there is a need to explore a simple and novel approach. There have been many impressive research efforts by a number of authors to analyze microstrip lines for capacitance and characteristic impedance using diverse approaches. The finite element method [2, 3], variational method [4], equivalent electrode [5], conformal method [6], spectral analysis [7], and a host of others have all been used till date. Fusco et al [8] and Shadare et al [9] have applied Markov chain for static field analysis and microstrip line analysis respectively. The drive to investigate a novel approach for the analysis of microstrip lines other than those already widely reported in the literature is the motivation for this work.

2. ABSORBING MARKON CHAIN

Classical Monte Carlo methods (such as the fixed random walk, the floating random walk, and the Exodus method) can only be used to calculate the potential at one point at a time. This limitation is overcome by the Monte Carlo Markov chain which is capable of whole field computation [10].

Suppose that the Monte Carlo Markov chain is to be applied in solving Laplace's equation

$$\nabla^2 V = 0 \quad \text{in region R} \tag{1}$$

subject to the Dirichlet boundary condition

$$V = V_p \text{ on boundary B} \tag{2}$$

A mesh is formed by dividing the region R using suitable step-size. Equation (1) is replaced by its finite difference equivalent as follows:

$$V(x.y) = p_{x+}V(x+\Delta, y) + p_{x-}V(x-\Delta, y) + p_{y+}V(x, y+\Delta) + p_{y-}V(x, y-\Delta)$$

(3)

where $p_{x+} = p_{x-} = p_{y+} = p_{y-} = \frac{1}{4}$ (4)

A Markov chain is defined as a sequence of random variables $X^{(0)}, X^{(1)},..., X^{(n)}$, where the probability distribution of $X^{(n)}$ is determined by the probability distribution $X^{(n-1)}$ [10, 11]. A Markov Chain is a process evolving in time that remembers only the most recent past and whose conditional probability distributions are time invariant. It is random and memoryless. The Markov chains of interest to us are discrete-state, discrete-time Markov chains. The transition probability P_{ij} is the probability that a random-walking particle at node i moves to node j. It is expressed by the Markov property

$$P_{ij} = P\left(x_{n+1} = j \mid i = x_0, x_1, ..., x_n\right) = P\left(x_{n+1} = j \mid i = x_n\right), \quad i, j \in X, n = 0, 1, 2, ... \qquad (5)$$

The Markov chain is characterized by its transition probability **P,** defined by

$$\mathbf{P} = \begin{bmatrix} P_{00} & P_{01} & P_{02} & \cdots \\ P_{10} & P_{11} & P_{12} & \cdots \\ P_{20} & P_{21} & P_{22} & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix} \qquad (6)$$

P is a stochastic matrix, meaning that the sum of the elements in each row is unity, that is,

$$\sum_{j \in X} P_{ij} = 1, \quad i \in X \qquad (7)$$

In our case, the Markov chain is assumed to be the random walk, and the states are the grid nodes. If we assume that there are n_f free (or non-absorbing) nodes and n_p fixed (absorbing) nodes, the size of the transition matrix **P** is n, where

$$n = n_f + n_p \qquad (8)$$

If the absorbing nodes are numbered first and the non-absorbing states are numbered last, the $n \times n$ transition matrix becomes

$$\mathbf{P} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{R} & \mathbf{Q} \end{bmatrix} \qquad (9)$$

where the $n_f \times n_p$ matrix **R** represents the probabilities of moving from non-absorbing nodes to absorbing ones; the $n_f \times n_p$ matrix **Q** represents the probabilities of moving from one non-absorbing node to another; $\mathbf{I}$ is the identity matrix representing transitions between the absorbing nodes $(P_{ii} = 1 \; and \; P_{ij} = 0)$; and **0** is the null matrix showing that there are no transitions from absorbing to non-absorbing nodes. From equation (4)

$$Q_{ij} = \begin{cases} \frac{1}{4}, & if \; i \; is \; directly \; connected \; to \; j \\ 0, & if \; i = j \; or \; i \; is \; not \; directly \; connected \; to \; j \end{cases} \qquad (10)$$

The same applies to R_{ij} except that j is an absorbing node. For any absorbing Markov chain, $\mathbf{I} - \mathbf{Q}$ has an inverse. This is usually referred to as the fundamental matrix

$$\mathbf{N} = (\mathbf{I} - \mathbf{Q})^{-1} \qquad (11)$$

where N_{ij} is the average number of times the random-walking particle starting from node i passes through node j before being absorbed. The absorption probability matrix **B** is

$$\mathbf{B} = \mathbf{N}\mathbf{R} \qquad (12)$$

where R_{ij} is the probability that a random-walking particle originating from a non-absorbing node i will end up at the absorbing node j. **B** is an $n_f \times n_p$ matrix and is stochastic, similar to the transition probability matrix, that is

$$\sum_{j=1}^{n_p} B_{ij} = 1, \quad i = 1, 2, ..., n_f \qquad (13)$$

If $\mathbf{V}_f$ and $\mathbf{V}_p$ contain potentials at the free and fixed nodes respectively, then

$$\mathbf{V}_f = \mathbf{B}\mathbf{V}_p \qquad (14)$$

3. PROBLEM FORMULATION

The microstrip line was divided into equal halves to take advantage of symmetry and only one half was analysed. The half-sized microstrip line (Fig. 1) was first analysed without the dielectric substrate and the procedure repeated for the case with the dielectric substrate. For the two cases, an artificial shielded height was introduced such that a finite domain was formed. The domain was then discretized into a number of grids (free and fixed nodes) using carefully selected step-size. The grid nodes are generated using the parameters shown in Table 1. Matrices are then developed from the grid based on probability and interactions of the random walking particles within the free nodes and between the free nodes and the absorbing nodes.

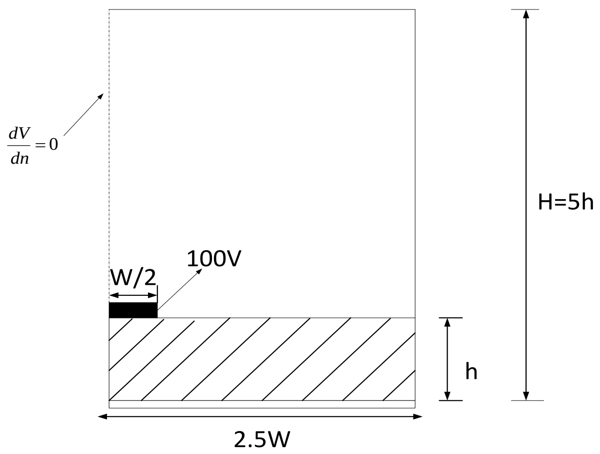

Figure. 1 Half sized microstrip structure

Table 1 shows the detailed parameters of the microstrip line that are used for our simulations with their explicit pictorial representation shown in Figure 1.

Table 1: Parameters of microstrip line

Parameter	Value
h	1.0cm
W	1.6cm
V_d	100V
H	5h
Size of dielectric	5W
Dielectric constant for alumina	9.6
Step size (Δ)	0.1 and 0.2cm

Case 1: Microstrip without the dielectric in place

A matrix is generated for **Q** that represents the probabilities of moving from one non-absorbing node to another. The transient probabilities are determined using equation (4) and on the line of symmetry, the condition $\frac{\partial V}{\partial n} = 0$ is imposed. The finite difference equivalent for line of symmetry is given by:

$$V_0 = p_{x+}V_1 + p_{y+}V_3 + p_{y-}V_4 \quad (15)$$

where $p_{x+} = \frac{1}{2},\ p_{y+} = p_{y-} = \frac{1}{4}$ (16)

The probabilities of the particles lying on the line of symmetry are determined using equation (16).

Case 2: Microstrip with the dielectric in place

Similarly, a matrix **Q** that represents the probabilities of moving from one non-absorbing node to another is generated for this case. The boundary condition $D_{1n} = D_{2n}$ is imposed. Consider the finite difference equivalent of the boundary condition at the interface given as:

$V_o = p_{x+}V_1 + p_{x-}V_2 + p_{y+}V_3 + p_{y-}V_4$ (17) where the transient probabilities are given by:

$$p_{x+} = p_{x-} = \frac{1}{4},\ p_{y+} = \frac{\varepsilon_1}{2(\varepsilon_1 + \varepsilon_2)},\ p_{y-} = \frac{\varepsilon_2}{2(\varepsilon_1 + \varepsilon_2)} \quad (18)$$

where in our case, ε_1 = dielectric constant of air and

ε_2 =dielectric constant of alumina. So, the probabilities and interactions of the random walking particles within the free nodes are determined as in case 1 except that particles beginning at the interface and moving to any immediate nodes have their probabilities determined by equation (18).

4. EVALUATION OF CHARACTERISTIC IMPEDANCE

(a). Evaluate the values of potential at free nodes as in Equation (14) for the case without the dielectric substrate

(b). Determine the value of the resultant charge, q [12] from Equation (21) and Figure 2.

$$q = \varepsilon_o\left[\sum \varepsilon_{ri}V_i \text{ for nodes i on external rectangle GHJMP}\right] - \varepsilon_o\left[\sum \varepsilon_{ri}V_i \text{ for nodes i on internal rectangle KLN}\right] \quad (19)$$

Note: corners such as J are not counted and corners such as L are counted twice

(c). Then, evaluate the capacitance C_o without the dielectric in place $C_o = \frac{4q_o}{V_d}$, where q_o is the resultant charge when the dielectric is not in place and V_d is the potential on the conductor

(d). Repeat steps (a) and (b) (with the dielectric substrate in place) and evaluate $C = \frac{4q}{V_d}$, where q is the resultant charge with the dielectric in place and V_d is the potential on the conductor

(e). Finally, calculate $Z_o = \frac{1}{u\sqrt{CC_o}}$, where $u = 3\times10^8$ m/s

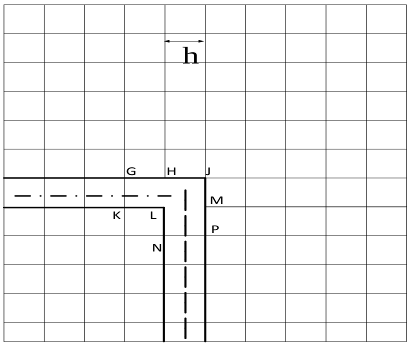

Figure 2. Rectangular shield around the strip conductor

5. RESULTS AND DISCUSSION

After we carried out our analysis, we obtained the following results. We compared our result with those obtained using analytical method and Finite Element Method as detailed in the Table 2. In the table, some results are not shown as they are not a requirement for obtaining the characteristic impedance through the analytical method

Table 2: Analytical result versus Simulation result

	Analytical Result	Finite element method	Markov chain Step size (0.2)	Markov chain Step size (0.1)
Capacitance without dielectric(F/m)	-	3.543×10^{-11}	4.1486×10^{-11}	3.7473×10^{-11}
Capacitance with dielectric (F/m)	-	2.2462×10^{-10}	2.4448×10^{-10}	2.4780×10^{-10}
Characteristic Impedance(ohms)	38.77	37.3653	33.0985	34.5916

Table 2 shows the analytical result and the finite element method compared with the simulation results. The capacitance per unit length without the dielectric substrate is as expected less than the capacitance per unit length with the dielectric substrate. The characteristic impedance obtained in the simulation shows close agreement with that obtained from analytical and finite element methods as the step-size is further reduced.

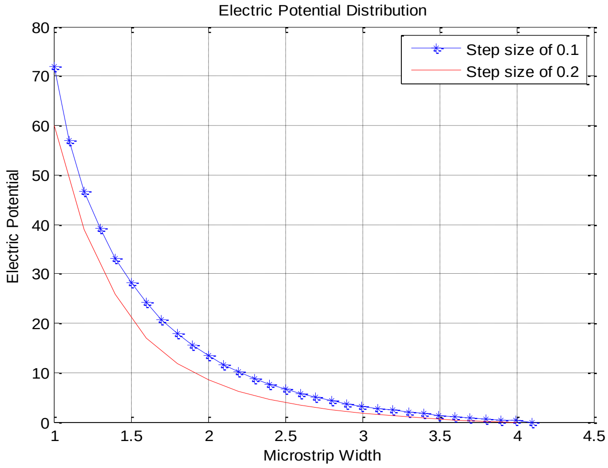

Figure 3. Potential distribution along the air-dielectric interface for the half-sized microstrip line

The Figure 3 shows the potential distribution along the air-dielectric interface for the half-sized microstrip line. From Figure 3, it is obvious that the electric potential along the dielectric interface decreases exponentially. It is zero at the point where the interface coincides with an absorbing node.

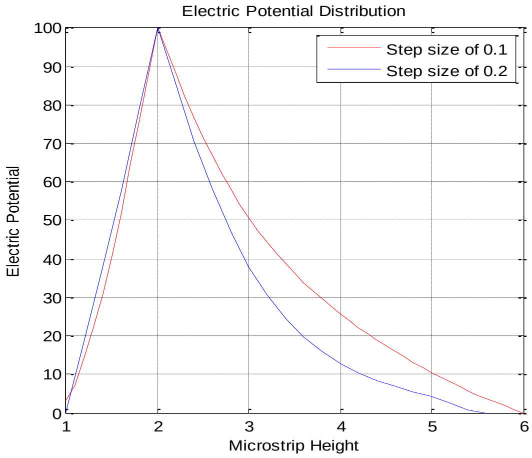

Figure. 4 Potential distribution along the line of symmetry for the half-sized microstrip line

The Figure 4 shows potential distribution along the line of symmetry for the microstrip line case with and without the dielectric substrate. The graph is peaked at the nodes where the strip conductor exists.

6. CONCLUSION

This research work analyzes microstrip lines using Markov Chain Monte Carlo method. The characteristic impedance value for the parameters of the microstrip line was obtained and compared closely with that obtained from analytical and finite element methods. Plots for the potential distribution along the dielectric-air interface and that along the line of symmetry were also obtained. It is hoped that this work will ease some identified and conceivable challenges that are associated with other methods previously deployed to the analysis of microstrip lines.

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