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Theories of Composite Plates and Numerical Methods Used on Bending and Buckling of Laminated Plates

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ABSTRACT

A comprehensive literature review on different theories of laminated plates have been reviewed and discussed thoroughly. It has been found that there are two main theories of laminated plates which are known as linear and nonlinear theories. The two theories are depending on the magnitude of deformation resulting from loading the given plates. The difference between the two theories is that the deformations are small in the linear theory, whereas they are finite or large in the nonlinear theory.

In comparisons between FEM and different numerical methods it has been found that FEM can be considered of acceptable accuracy, and can also be applied to different complicated geometries and shapes.

Keywords: *Theories of laminates, linear and nonlinear, numerical methods, finite element method, small and large deformations.*

1. Developments in the Theories of Laminated Plates

From the point of view of solid mechanics, the deformation of a plate subjected to transverse and / or in plane loading consists of two components: flexural deformation due to rotation of cross – sections, and shear deformation due to sliding of section or layers. The resulting deformation depends on two parameters: the thickness to length ratio and the ratio of elastic to shear moduli. When the thickness to length ratio is small, the plate is considered thin, and it deforms mainly by flexure or bending; whereas when the thickness to length and the modular ratios are both large, the plate deforms mainly through shear. Due to the high ratio of in – plane modulus to transverse shear modulus, the shear deformation effects are more pronounced in the composite laminates subjected to transverse and / or in plane loads than in the isotropic plates under similar loading conditions.

The three – dimensional theories of laminates in which each layer is treated as homogeneous anisotropic medium (see Reddy [1]) are intractable. Usually, the anisotropy in laminated composite structures causes complicated responses under different loading conditions by creating complex couplings between extensions and bending, and shears deformation modes. Expect for certain cases, it is inconvenient to fully solve a problem in three dimensions due to the complexity, size of computation, and the production of unnecessary data specially for composite structures.

Many theories which account for the transverse shear and normal stresses are available in the literature (see, for example Mindlin [2]). These are too numerous to review here. Only some classical papers and those which constitute a background for the present thesis will be considered. These theories are classified according to Phan and Reddy [3] into two major classes on the basis of the assumed fields as: (1) stress based theories, and (2) displacement based theories. The stress – based theories are derived from stress fields which are assumed to vary linearly over the thickness of the plate:

$$\sigma_i = \frac{M_i}{\left(h^2/6\right)} \times \frac{z}{\left(h/2\right)} \qquad \left(i = 1,2,6\right) \tag{1}$$

(Where M_i is the stress couples, h is the plate thickness, and z is the distance of the lamina from the plate mid – plane).

The displacement – based theories are derived from an assumed displacement field as:

$$\begin{aligned} u &= u_\circ + z u_1 + z^2 u_2 + z^3 u_3 + \ldots\ldots \\ v &= v_\circ + z v_1 + z^2 v_2 + z^3 v_3 + \ldots\ldots \\ w &= w_\circ + z w_1 + z^2 w_2 + z^3 w_3 + \ldots\ldots \end{aligned} \tag{2}$$

Where: $u_\circ, v_\circ$ and $w_\circ$ are the displacements of the middle plane of the plate. The governing equations are derived using principle of minimum total potential energy. The theory used in the present work comes under the class of displacement – based theories. Extensions of these theories which include the linear terms in z in u, v and only the constant term in w, to account for higher – order variations and to laminated plates, can be found in the work of Yang, Norris and Stavsky [4] , Whitney and Pagano [5] and Phan and Reddy [3].

Based on different assumptions for displacement fields, different theories for plate analysis have been devised. These theories can be divided into three major categories, the individual layer theories (IL), the equivalent single layer (ESL) theories, and the three dimensional elasticity solution procedures. These categories are further divided into sub –

theories by the introduction of different assumptions. For example the second category includes the classical laminated plate theory (CLPT), the first order and higher order shear deformation theories (FSDT and HSDT) as stated in Refs.{ [6]–[9]}.

In the individual layer laminate theories, each layer is considered as a separate plate. Since the displacement fields and equilibrium equations are written for each layer, adjacent layers must be matched at each interface by selecting appropriate interfacial conditions for displacements and stresses. In the ESL laminate theories, the stress or the displacement field is expressed as a linear combination of unknown functions and the coordinate along the thickness. If the in – plane displacements are expanded in terms of the thickness co – ordinate up to the n^{th} power, the theory is named n^{th} order shear deformation theory. The simplest ESL laminate theory is the classical laminated plate theory (CLPT). This theory is applicable to homogeneous thin plates (i.e. the length to thickness ratio $a / h > 20$). The classical laminated plate theory (CLPT), which is an extension of the classical plate theory (CPT) applied to laminated plates was the first theory formulated for the analysis of laminated plates by Reissner and Stavsky [10] in 1961 , in which the Kirchhoff and Love assumption that normal to the mid – surface before deformation remain straight and normal to the mid – surface after deformation is used (see fig. 1), but it is not adequate for the flexural analysis of moderately thick laminates. However, it gives reasonably accurate results for many engineering problems i.e. thin composite plates, as stated by Srinivas and Rao [11], Reissner and Stavsky [10]. This theory ignores the transverse shear stress components and models a laminate as an equivalent single layer. The classical laminated plate theory (CLPT) under – predicts deflections as proved by Turvey and Osman [12], [13], and [14] and Reddy [1] due to the neglect of transverse shearstrain. The errors in deflection are even higher for plates made of advanced filamentary composite materials like graphite – epoxy and boron – epoxy whose elastic modulus to shear modulus ratios are very large (i.e. of the order of 25 to 40 , instead of 2.6 for typical isotropic materials). However, these composites are susceptible to thickness effects because their effective transverse shear moduli are significantly smaller than the effective elastic modulus along the fiber direction. This effect has been confirmed by Pagano [15] who obtained analytical solutions of laminated plates in bending based on the three – dimensional theory of elasticity. He proved that classical laminated plate theory (CLPT) becomes of less accuracy as the side to thicknessratio decreases. In particular, the deflection of a plate predicted by CLPT is considerably smaller than the analytical value for side to thickness ratio less than 10. These high ratios of elastic modulus to shear modulus render classical laminate theory as inadequate for the analysis of composite plates. In the first order shear deformation theory (FSDT) , the transverse planes , which are originally normal and straight to the mid – plane of the plate , are assumed to remain straight but not necessarily normal after deformation , and consequently shear correction factors are employed in this theory to adjust the transverse shear stress , which is constant through thickness (see fig. 1). Recently Reddy [1] and Phan and Reddy [3] presented refined plate theories that used the idea of expanding displacements in the powers of thickness coordinate. The main novelty of these works is to expand the in – plane displacements as cubic functions of the thickness coordinate, treat the transverse deflection as a function of the x and y coordinates, and eliminate the functions u_2 , u_3, v_2 and v_3 from equation (2) by requiring that the transverse shear stress be zero on the bounding planes of the plate. Numerous studies involving the application of the first – order theory to bending, vibration and buckling analyses can be found in the works of Reddy [16], and Reddy and Chao [17].

In order to include the curvature of the normal after deformation, a number of theories known as higher – order shear deformation theories (HSDT) have been devised in which the displacements are assumed quadratic or cubic through the thickness of the plate. In this aspect, a variationally consistent higher – order theory which not only accounts for the shear deformation but also satisfies the zero transverse shear stress conditions on the top and bottom faces of the plate and does not require correction factors was suggested by Reddy [1]. Reddy's modifications consist of a more systematic derivation of displacement field and variationally consistent derivation of the equilibrium equations. The refined laminate plate theory predicts a parabolic distribution of the transverse shear stresses through the thickness, and requires no shear correction coefficients.

In the non – linear analysis of plates considering higher – order shear deformation theory (HSDT), shear deformation has received considerably less attention compared with linear analysis. This is due to the geometric non – linearity which arises from finite deformations of an elastic body and which causes more complications in the analysis of composite plates. Therefore, fiber – reinforced material properties and lamination geometry have to be taken into account. In the case of anti – symmetric and unsymmetrical laminates, the existence of coupling between stretching and bending complicates the problem further. Non – linear solutions of laminated plates using higher – order theories have been obtained through several techniques, i. e. perturbation method as in Ref.[18], finite element method as in Ref.[19], the increment of lateral displacement method as in Ref.[20],and the small parameter method as in Ref.[21].

2. Numerical Techniques

Several numerical methods could be used in this study, but the main ones are finite difference method (FDM), dynamic relaxation coupled with finite difference method (DR), and finite element method (FEM).

In the finite difference method, the solution domain is divided into a grid of discrete points or nodes. The partial differential equation is then written for each node and its derivatives are replaced by finite divided differences. Although such point – wise approximation is conceptually easy to understand, it becomes difficult to apply for system with irregular geometry, unusual boundary conditions, and heterogeneous composition.

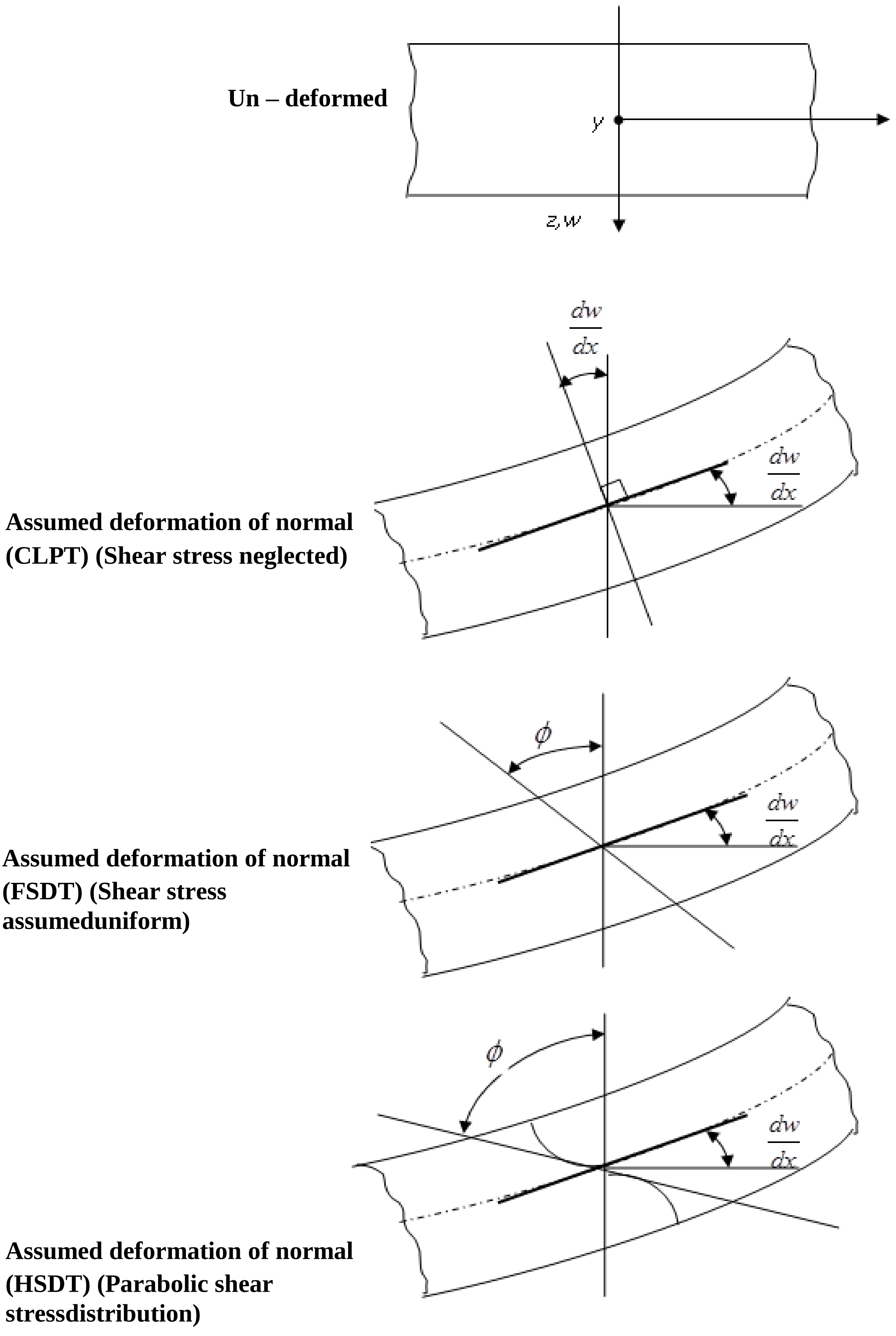

Fig. (1) Assumed Deformation of the Transverse Normal in Various Displacement BasePlate Theories.

The DR method was first proposed in 1960th; see Rushton [22], Cassel and Hobbs [23], and Day [24]. In this method, the equations of equilibrium are converted to dynamic equations by adding damping and inertia terms. These are then expressed in finite difference form and the solution is obtained through iterations. The optimum damping coefficient and the time increment used to stabilize the solution depend on the stiffness matrix of the structure, the applied load, the boundary conditions and the size of mesh used.

In the present work, a numerical method known as finite element method (FEM) is used. It is a numerical procedure for obtaining solutions to many of the problems encountered in engineering analysis. It has two primary subdivisions. The first utilizes discrete elements to obtain the joint displacements and member forces of a structural framework. The second uses the continuum elements to obtain approximate solutions to heat transfer, fluid mechanics, and solid mechanics problem. The formulation using the discrete element is referred to as matrix analysis of structures and yields results identical with the classical analysis of structural frameworks. The second approach is the true finite element method. It yields approximate values of the desired parameters at specific points called nodes. A general finite element computers program, however, is capable of solving both types of problems and the name" finite element method" is often used to denote both the discrete element and the continuum element formulations.

The finite element method combines several mathematical concepts to produce a system of linear and non – linear equations. The number of equations is usually very large, anywhere from 20 to 20,000 or more and requires the computational power of the digital computer.

It is impossible to document the exact origin of the finite element method because the basic concepts have evolved over a period of 150 or more years. The method as we know it today is an outgrowth of several papers published in the 1950th that extended the matrix analysis of structures to continuum bodies. The space exploration of the 1960th provided money for basic research, which placed the method of a firm mathematical foundation and stimulated the development of multi – purpose computer programs that implemented the method. The design of airplanes, unmanned drones, missiles, space capsules, and the like, provided application areas.

The finite element method (FEM) is a powerful numerical method, which is used as a computational technique for the solution of differential equations that arise in various fields of engineering and applied sciences. The finite element method is based on the concept that one can replace any continuum by an assemblage of simply shaped elements, called finite elements with well-defined force, displacement, and material relationships. While one may not be able to derive a closed – form solution for the continuum, one can derive approximate solutions for the element assemblage that replaces it. The approximate solutions or approximation functions are often constructed using ideas from interpolation theory, and hence they are also called interpolation functions. For more details refer to Refs. {[25] – [27]}.

In a comparison between the finite element method (FEM) and dynamic relaxation method (DR), Aalami [28], Turvey and Osman {[12] – [14]} and Osama Mohammed Elmardi Suleiman {[29] – [35]}found that the computer time required for the finite element method is eight times greater than for DR analysis, whereas the storage capacity for FEM is ten times or more than that for DR analysis. This fact is supported by Putcha and Reddy [19], and Turvey and Osman {[12] –[14]} who noted that some of the finite element formulations require large storage capacity and computer time. Hence due to the large computations involved in the present study, the finite element method (FEM) is considered more efficient than the DR method. In another comparison, Aalami [28] found that the difference in accuracy between one version of FEM and DR may reach a value of more than 15 % in favor of FEM. Therefore, the FEM can be considered of acceptable accuracy. The apparent limitation of the DR method is that it can only be applied to limited geometries, whereas the FEM can be applied to different intricate geometries and shapes.

3. Conclusion

It was found that there are two theories of composite laminated plates which are known as small and large deformation theories. The difference between the two theories is that, the deflections are small in the first mentioned theory and large in the second theory.

It was also found that, the selection of a suitable numerical method is dependent on the degree of complication of the laminates and the accuracy required.

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