

# The Use of First Order Polynomial with Double Scalar Quantization for Image Compression

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## ABSTRACT

*In this paper, the proposed technique attempts to exploit the first order polynomial model effectively along with utilizing the two stages of scalar quantizer to enhance the system efficiency. The test results of the suggested method are promising especially with the identical base model compared with the traditional and separate base models.*

**Key Words:** Image compression, first order polynomial model, and two stage scalar quantizer.

## 1. INTRODUCTION

Data compression is an old and eternally new research area, that implicitly means reduce expensive resources, such as storage memory or transmission bandwidth due to removing redundancy(s) [1,2]. In other words, data compression is still an interesting topic also because of the number of files/data keep growing exponentially. There are many examples of fast growing digital data. The first example is in radiography and medical imaging, where hospitals and clinic environments are rapidly moving towards computerization that means digitization, processing, storage and transmission of medical image. And there are two reasons why these data keep growing exponentially, firstly because of the growth of the patients that need to be scanned, secondly is the conversion of archived film medical images. The second example of the growing digital data is in the oil and gas industry that has been developing what's known as the 'digital oilfield', where sensors monitoring activity at the point of exploration and the wellhead connect to information system at headquarters and drive operation and exploration decisions in real time, that means more data produce each data, also there are more applications that produce data exponentially such as broadcast, media, entertainment [1]. Those digitized data have heightened the challenge for ways to manage organize the data (compression, storage and transmission). This purpose is emphasized, for example, by non compression of raw image with size  $512 \times 512$  pixels, each pixel is represented by 8 bits, contains 262 KB of data. Moreover, image size will be tripled if the image is represented in color. Furthermore, if the image is composed for a video that needs generally 25 frames per second for just a one second of color film, requires approximately 19 megabytes of memory. So, a memory with 540 MB can only store only about 30 seconds of film. Thus, data compression process is really obvious to represent data into smaller size possible, therefore less storage capacity will be needed and data transmission will be faster than uncompressed data [1].

A vast amount of work had been done to improve the performance of image data compression techniques, with tools such as run length coding, predictive coding, transform coding, block truncation coding and vector quantization. Usually there is a trade off between the possible compression ratio, degradation and complexity. However, some efforts

go beyond this and have constructed hybrid techniques based on combining image coding techniques, in order to increase the efficiency of performance [2]. On the other hand modern techniques remain in development, for example fractal compression and predictive coding, which may in the near future be adopted as one of the standard image coding techniques [2].

Polynomial coding is a modern image compression technique based on modelling concept to remove the spatial redundancy embedded within the image effectively. The basic idea of polynomial coding is the utilization of mathematical model to represent each nonoverlapping partitioning block with a small number of coefficients of low error (residual) [3].

This paper is concerned with improving the polynomial techniques performance of linear base namely first order base, using a two stage multiple description scalar quantizer. The rest of paper organized as follows, section 2 describes the first order polynomial model, separate and identical proposed models, the results for the proposed system and the conclusions, is given in sections 4,5,6 and 7, respectively.

## 2. Two Stage Multiple Description Scalar Quantizer

Multiple description (*MD*) coding addresses the problem of packet loss in a communication network by sending several descriptions of the source over different paths such that the reconstruction quality improves with the number of received descriptions [4]. The *MD* scalar quantizer (*MDSQ*) developed in [5] is applied to the *MD* predictive coding of video, where each description uses a separate predictive encoder with a quantizer equivalent to a side quantizer in the *MDSQ*. To maintain the necessary offset between the side quantizers, the predictor output is quantized by additional quantizer, which renders the prediction sub optimal [4]. In the two stage *MDSQ*, a source sample is quantized by two uniform quantizers whose cells are staggered by an offset of  $q/2$ , where  $q$  is the quantization step corresponding to the first stage uniform quantizer. In other words, the combination of the two quantizers yields a joint quantizer with step size  $q/2$ . The joint quantizer error is further quantized by a finer second stage uniform quantizer [4], figure (1) shows *MDSQ* predictive encoding.

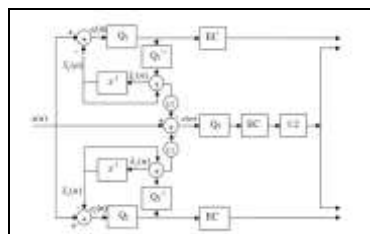


Fig (1): *MDSQ* predictive encoder [4].

The idea of *MDSQ* is illustrated in figure above, consider a source input  $x(n)$ , where each descriptor uses a separate scalar quantizer based predictive coding model encoder. The average of reconstructions  $\bar{x}_1(n)$  and  $\bar{x}_2(n)$  is used to refined reconstruction of the source. The corresponding error  $e(n)$  is quantized by another scalar quantizer and the result is evenly split into the two descriptions, as in the *MDSQ*.  $Q_1$ ,  $Q_2$  and  $Q_3$  are all uniform scalar quantizer and their outputs are entropy coded [4].

## 3. First Order Polynomial Coding

This approach corresponds to the basic linear model adopted to compress images by determining the coefficients of the mathematical model that enable us to crate the estimate image and straightly find the residual, based on exploiting the interpixel redundancy [3].

The implementation of the linear polynomial based compression system is explained in the following steps [6,7]:

**Step 1:** Load the input uncompressed gray image  $I$  of *BMP* format of size  $N \times N$ .

**Step 2:** Partition the image ( $I$ ) into nonoverlapped blocks of fixed size  $n \times n$ , such as  $(4 \times 4)$  or  $(8 \times 8)$  then compute the coefficients according to equations (1-3).

$$a_0 = \frac{1}{n \times n} \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} I(i, j) \dots \dots \dots (1)$$

$$a_1 = \frac{\sum_{i=0}^{n-1} \sum_{j=0}^{n-1} I(i, j) \times (j - x_c)}{\sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (j - x_c)^2} \dots \dots \dots (3)$$

$$a_2 = \frac{\sum_{i=0}^{n-1} \sum_{j=0}^{n-1} I(i, j) \times (i - y_c)}{\sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (i - y_c)^2} \dots \dots \dots (2)$$

Where  $a_0$  coefficient corresponds to the mean (average) of block of size  $(n \times n)$  of original image  $I$ . The  $a_1$  and  $a_2$  coefficients represent the ratio of sum pixel multiplied by the distance from the center to the squared distance in  $i$  and  $j$  coordinates respectively, and the  $(j-x_c)$  and  $(i-y_c)$  corresponds to measure the distance of pixel coordinates to the block center  $(x_c, y_c)$ [3].

$$x_c = y_c = \frac{n-1}{2} \dots \dots \dots (4)$$

**Step 3:** Apply uniform scalar quantization/dequantization of the computed polynomial approximation coefficients, where each coefficient is quantized using different quantization step.

$$a_0Q = \text{round}\left(\frac{a_0}{QS_{a0}}\right) \rightarrow a_0D = a_0Q \times QS_{a0} \dots \dots \dots (5)$$

$$a_1Q = \text{round}\left(\frac{a_1}{QS_{a1}}\right) \rightarrow a_1D = a_1Q \times QS_{a1} \dots \dots \dots (6)$$

$$a_2Q = \text{round}\left(\frac{a_2}{QS_{a2}}\right) \rightarrow a_2D = a_2Q \times QS_{a2} \dots \dots \dots (7)$$

Where  $a_0Q, a_1Q, a_2Q$  are the polynomial quantized values,  $QS_{a0}, QS_{a1}, QS_{a2}$  are the quantization steps of the polynomial coefficients, and  $a_0D, a_1D, a_2D$  are polynomial dequantized values.

**Step 4:** Determine the predicted image value  $\tilde{I}$  using the dequantized polynomial coefficients for each encoded block representation:

$$\tilde{I} = a_0D + a_1D(j - x_c) + a_2D(i - y_c) \dots \dots \dots (8)$$

**Step 5:** Find the residual or prediction error as difference between the original  $I$  and the predicted one  $\tilde{I}$ .

$$Re\ s(i, j) = I(i, j) - \tilde{I}(i, j) \dots \dots \dots (9)$$

**Step 6:** Perform scalar uniform quantization\dequantization of the residual part as in Step 3 above.

$$Re\ sQ = \text{round}\left(\frac{Re\ s}{QS_{Re\ s}}\right) \rightarrow Re\ sD = Re\ sQ \times QS_{Re\ s} \dots \dots \dots (10)$$

**Step 7:** Apply Huffman coding techniques to remove the coding redundancy that embedded between the quantized values of the residual and the polynomial coefficients.

**Step 8:** To reconstruct the compressed image, the decoder, adds the estimated image to the dequantized decoded residual one.

$$\hat{I}(i, j) = \tilde{I}(i, j) + Re\ sD(i, j) \dots \dots \dots (11)$$

### 4. Two Stage Scalar Quantizer Separate First Order Polynomial Coding

The idea relies on utilizing the linear polynomial coding twice along with the two stage scalar quantizer idea. The proposed system is depicted in figure (2), the following steps are applied.

**Step 1:** Load the original uncompressed gray image  $I$  of  $BMP$  format of size  $N \times N$ .

**Step 2:** Apply the first order polynomial coding techniques twice, or simply two times separately, where for each time starts by partitioning the image  $I$  into fixed blocks of size  $n \times n$  and computing the coefficients of each block, followed by creating the predicted  $\tilde{I}$  image and lastly the residual quantized/dequantized calculated (see section (3)) in other words, by applying the separate polynomial coding of the input image  $I$ , we have two predicted and residual images ( $\tilde{I}_1$  &  $\tilde{I}_2$ ) and ( $Res_1$  &  $Res_2$ ) of model1 and model2 respectively.

**Step 3:** Find the average of the two dequantized residual images from the step above, that added to find the reconstructed image as in equation (11).

$$Re\ sDAv(i, j) = (Re\ sD1(i, j) + Re\ sD2(i, j)) / 2 \dots\dots\dots (12)$$

Where  $Re\ sD1$ ,  $Re\ sD2$  are the dequantized residual images of two separate linear models.

**Step 4:** Find the second residual image as a difference between the original image and the reconstructed from Step 3 above, which also quantized using the scalar base that corresponds to the second stage quantizer and reconstruct the compressed image according to equation (11). As mentioned previously the Huffman coding techniques utilized to remove the coding redundancy.

### 5. Two Stage Scalar Quantizer Identical First Order Polynomial Coding

This technique is identical to the two stage scalar quantizer separate linear polynomial coding discussed above (see section 4), but with using linear polynomial once to overcome the exhausted bytes of computing the coefficients of second separate linear model where the following steps are applied. Figure (3) illustrates the proposed techniques steps.

**Step 1:** Load the original uncompressed gray image  $I$  of  $BMP$  format of size  $N \times N$ .

**Step 2:** Apply the linear polynomial coding techniques, that starts by partitioning the image  $I$  into fixed blocks of size  $n \times n$  and computing the coefficients of each block, followed by creating the predicted  $\tilde{I}$  image and lastly the residual calculated  $Res$  (see section (3)).

**Step 3:** Perform scalar uniform quantization\dequantization twice of the residual part resultant from step above. Where each residual image is quantized using different quantization step.

$$Re\ sQ1 = round(\frac{Re\ s}{Q1SRe\ s}) \rightarrow Re\ sD1 = Re\ sQ1 \times Q1SRe\ s \dots\dots\dots (13)$$

$$Re\ sQ2 = round(\frac{Re\ s}{Q2SRe\ s}) \rightarrow Re\ sD2 = Re\ sQ2 \times Q2SRe\ s \dots\dots\dots (14)$$

Where  $Re\ sQ1, Re\ sQ2$  are the residual quantized images,  $Q1SRe\ s$ ,  $Q2SRe\ s$  are the quantization steps of the residuals, and  $Re\ sD1$ ,  $Re\ sD2$  are the dequantized residual images.

**Step 4:** Use the two stage scalar quantizer discussed previously (see section (4) Steps 3&4), by simply finding the average of the two dequantized residual images according to equation (12) and applying the scalar quantizer of the second residual images, then reconstruct the compressed image. Also the symbol encoder/decoder adopted here is the Huffman coding techniques.

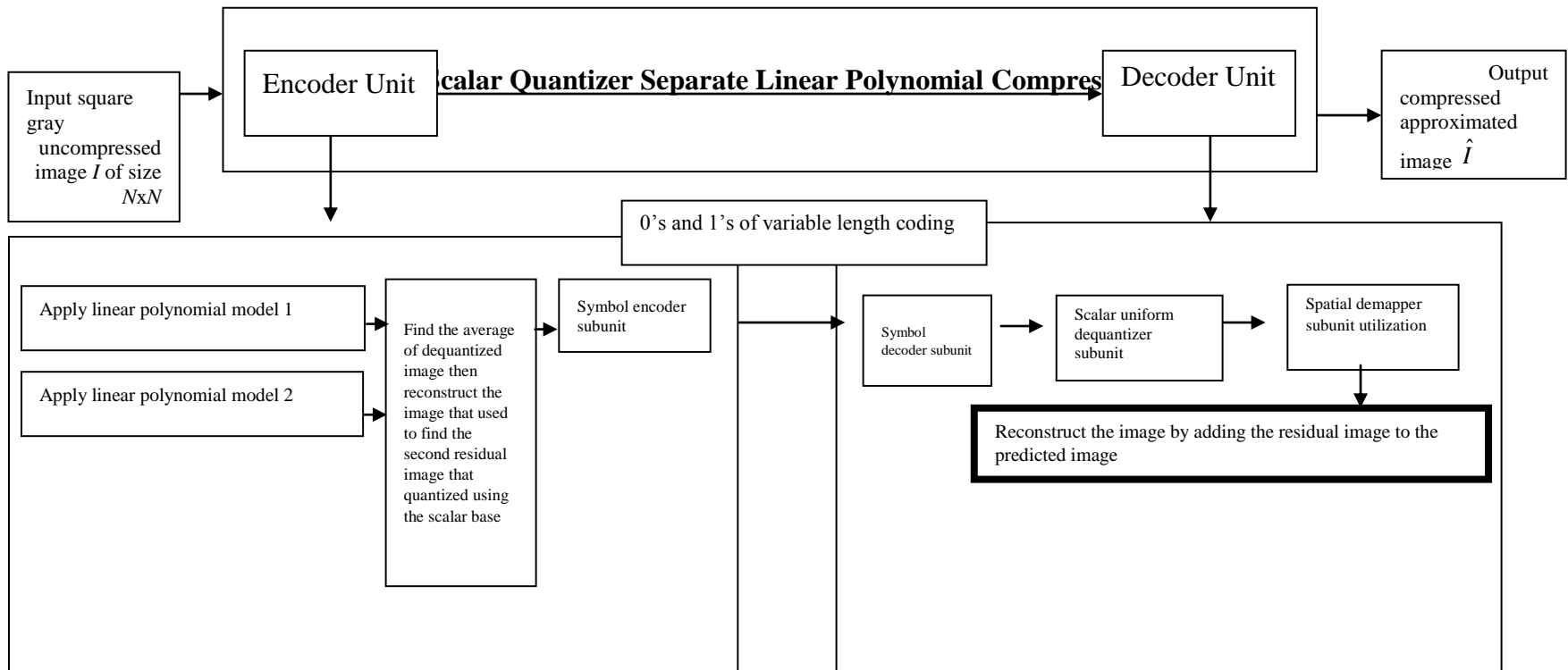


Fig. (2): Two stage Scalar Quantizer Separate First Order Polynomial Model

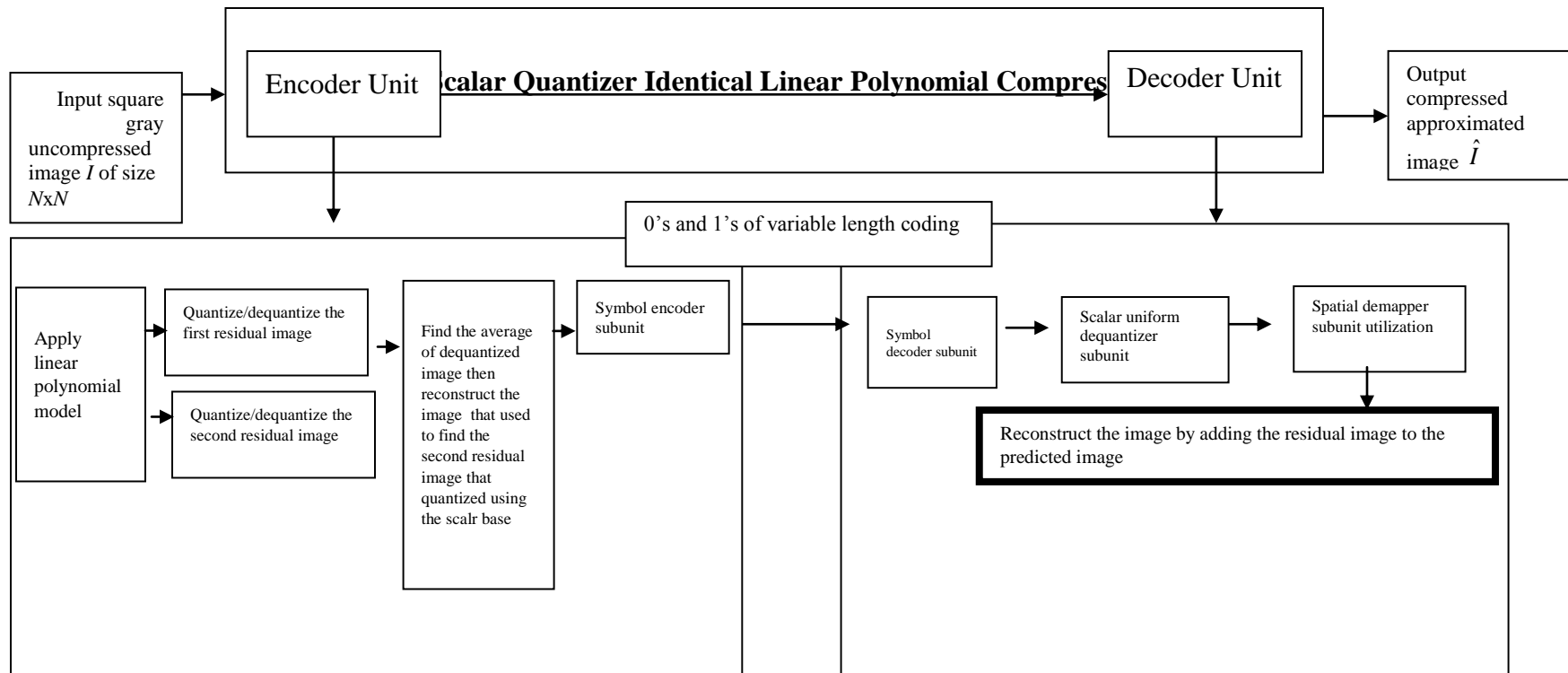


Fig. (3): Two stage Scalar Quantizer Identical First Order Polynomial Model.

## 6. Experimental Results

In order to test the performance of the polynomial coding techniques of linear/nonlinear base and comparing it with the proposed methods of two stage scalar quantizer, two standard gray (256 gray levels or 8 bits/pixel) square (256×256) images of varying details are utilized as shown in figure (4), also the compression ratio utilized along with the objective fidelity criteria of *PSNR* measure according to equations below.

$$CR = \frac{\text{Size of Original Image in Byte}}{\text{Size of Compressed Image Information in Byte}} \dots\dots\dots(15)$$

$$PSNR(dB) = 10 \log_{10} \left[ \frac{(\text{maximum gray scale of image})^2}{MSE} \right] \dots\dots\dots(16)$$

$$MSE(I, \hat{I}) = \frac{1}{N \times N} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} [\hat{I}(i, j) - I(i, j)]^2 \dots\dots\dots(17)$$



**Fig. (4): The tested images of size 256×256, gray scale images, (a) Lena and (b) Rose.**

The results shown in tables (1 & 2) illustrate the comparison between the traditional polynomial coding of first order model and proposed two stage scalar quantizer polynomial coding techniques of separate base, and identical base of two tested images using block sizes of 4x4. It is clear that the compression ratio and the quality of the decoded image affected by the quantization steps, where usually there is a trade off (inverse relation) between them, namely the compression ratio and the decoded image quality, where for low compression ratio attains the high *PSNR* value, and vice versa.

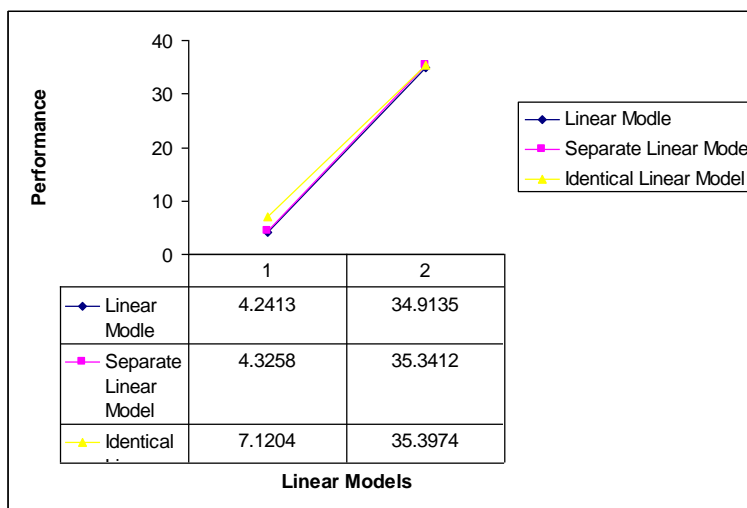
The results showed that the separate techniques of linear base is byte consumption since it would needs to compute the coefficients twice for each model independently that characterized by large residual images. Basically, the separate model followed the same behaviour as the traditional linear model, while the best performance in terms of compression ratio and *PSNR* achieved by utilizing the identical linear model, that due to using the linear model once to overcome the coefficients burden problem. Figure (5 a&b) summarizes the performance of the linear base techniques of the tested images. Lastly, as a spatial technique base the results vary according to image details or characteristics, where for a low image details, more compression ratio can be achieved compared to high image characteristic.

**Table (1): Comparison performance between polynomial coding and proposed techniques of two stages scalar quantizer of Lena image.**

Tested image	Linear Polynomial Coding with Qcoeff=1,2,2		Separate Linear with two stage SC Qcoeff1=1,2,2 Qcoeff2=1,3 ,3		Identical Linear with two stage SC Qcoeff1=1,2,2	
	Lena	Q Res=20		Q Res=20,60,20		Q Res=20,60,20
CR		PSNR	CR	PSNR	CR	PSNR
4.2413		34.9135	4.3258	35.3412	7.1204	35.3974
Q Res=50		Q Res=20,60,50		Q Res=20,60,50		
4.4667	30.0366	4.5868	33.3920	7.8317	33.2763	

**Table (2): Comparison performance between polynomial coding and proposed techniques of two stages scalar quantizer of Rose image.**

Tested image	Linear Polynomial Coding with Qcoeff=1,2,2		Separate Linear with two stage SC Qcoeff1=1,2,2 Qcoeff2=1,3 ,3		Identical Linear with two stage SC Qcoeff1=1,2,2	
	Rose	Q Res=20		Q Res=20,60,20		Q Res=20,60,20
CR		PSNR	CR	PSNR	CR	PSNR
4.3743		36.3577	4.7332	36.9302	8.3934	36.9394
Q Res=50		Q Res=20,60,50		Q Res=20,60,50		
4.4943	32.5447	4.8588	35.8171	8.7709	35.7451	



**Fig (5 a): Performance of linear model based of Lena image, using quantization polynomial coefficients of 1,2,2 and quantization residual of 20 of traditional linear model, quantization polynomial coefficients of 1,2,2, 1,3,3 and quantization residual of 20,60,20 of separate linear model, and quantization polynomial coefficients of 1,2,2 and quantization residual of 20,60,20 of identical linear model.**



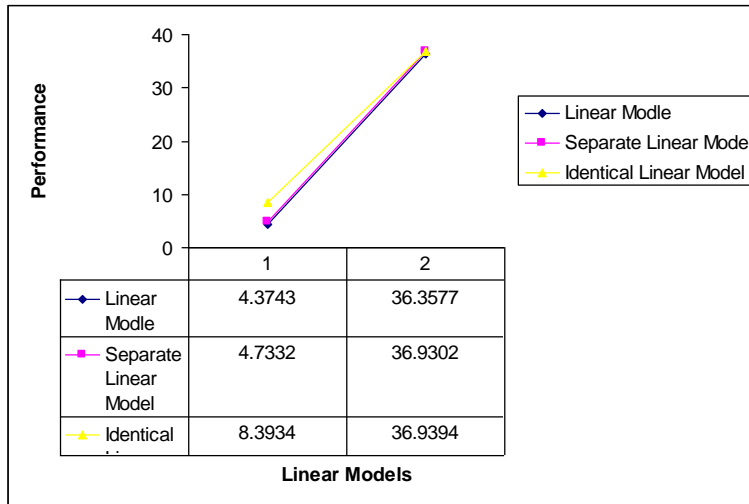


Fig (5 b): Performance of linear model based of Rose image, using quantization polynomial coefficients of 1,2,2 and quantization residual of 20 of traditional linear model, quantization polynomial coefficients of 1,2,2, 1,3,3 and quantization residual of 20,60,20 of separate linear model, and quantization polynomial coefficients of 1,2,2 and quantization residual of 20,60,20 of identical linear model.

## 7. Conclusions

From the test results of the proposed system, the following remarks are stimulated:

- 1- The polynomial coding is spatial technique based, where the method(s) is simple to implement and directly affected by the way of coding the coefficients and residual.
- 2- The proposed technique of identical first order base two stage scalar quantizer leads to improve the results compared to traditional polynomial coding and separate two stage scalar quantizer.

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