Study and Comparison of Different Plate Theory

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Abstract

The objective of this article is to present an investigation on the various theories and presented some classical and high order models, these models to use the new function f(z) for the static and dynamic analysis of plates (laminate, Composite and FGM) as well as those theories which have been developed in the literature to improve the evolution of the variation of the field of displacements in the thickness of the materials.

Key Words: *Plate Theories, High Order Theory, Laminate, Composite, FGM.*

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1. INTRODUCTION

The modeling of modern multi-layer structures with strong anisotropy (eg: low ratio of the transverse shear modulus of the core to the longitudinal elastic shear modulus of the skins in the case of sandwich structures) requires refined theories that take a good description of the transverse shears Nguyen et al [35]. Complete literature is available in the literature Noor et al [37]; Kapania et al [20]; Kant et al [21]; Carrera [7], on existing the different models of plate type.

The choice of functions and their different forms depends essentially on deformation theories which allow, on the one hand, transverse shear to be taken into account with the exception of Euler Bernoulli's theory, and on the other hand to approach it Shape of the distribution of the shear stresses through the thickness of the plate. In this article, a bibliographical study of the different theory of plates is presented. A bibliographic synthesis on the various models of plate theories that includes transverse shear, in addition to a comparison between classical theories and high order theories is presented.

2. Description of the plates

Beside linear structures (beams and frames) where one dimension dominates the other two, structures are often found in structures that extend over a flat or curved surface (two-dimensional structures). In this case, two dimensions dominate the third (thickness). If the surface is curved, we speak of hull or dome Fig.1.
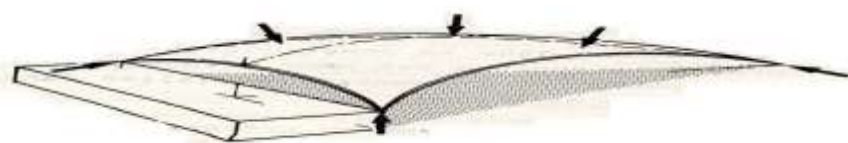

Figure.1. Shell or coupole.

If the surface is flat, one must distinguish between:

- The flat sails or loads acting essentially in the plane of the sail,

- The slabs, where the loads act essentially perpendicular to the surface. This type of structure is encountered very frequently in construction. In the building, this form of structure is even predominant. While in the bridges, we meet the slabs in the superstructure.

3. CLASSIQUE MODEL

These models are based on a linear distribution of displacements in the thickness Reissner et al **[38]**, Yang et al **[55]**, The deformations due to transverse shears are neglected, the normal remains straight and perpendicular to the mean surface after deformation.

3.1. The fundamental hypotheses of the theory of beams and plates

3.1.1. Principle of Saint venant

The principle of saint is: The stress at a point remote from the points of application of a system of forces depends only on the general resultant and the moment resulting from this system of forces, even if the distribution of the constraints is not the same, the solution found Will be valid if the loads are placed sufficiently far from the point of application.

3.1.2. Principle of N.Bernoulli Generalized

The hypothesis of Navier Bernoulli consists in assuming that the sections normal to the average fiber remain flat during the deformation of the beam to the plates. This hypothesis, which makes it possible to calculate the normal stresses due to bending moment, is well verified in the case of pure bending where the shear force is zero. On the other hand, in the case of simple bending with shear force, the sections do not remain Planes, but warp in the form of letter S very compressed. Similarly, when we study the torsion, we see that a non-circular section, with two symmetric axes, holds under the effect of a torsional torque a radial warp. The principle of Navier Bernoulli is based on the following observations:

- The warping of a section is always very small with respect to the dimensions of the section.

- The variation of the warpage, when passing from a section to an infinitely neighboring section, is always very small, not only with respect to the distance of the two infinitely neighboring sections.

- The principle of Navier Bernoulli amounts to neglecting shear and warping of the cross sections in the study of displacement and deformation of a beam element to plate.

It is rare to find a theory that would be applicable to all possible cases (composite, anisotropic, isotropic, large number of layers, sandwich stratification etc.) and to the different domains (static, dynamic and buckling), and Simple and easy and does not cost expensive in computing time. The oldest theory is that of Kirchoff Dhatt **[12]** which neglects the transverse shear effect. It can therefore only be applied to very thin structures. The first-order theory commonly associated with Mindlin **[26]** and Reissner **[37]**, which was one of the first to state its bases, takes into account the effects of transverse shear across the thickness. It leads, by the hypothesis of "straight sections remain straight" to a vector of constant transverse shear stresses in the thickness, in contradiction with a quadratic representation conventionally obtained for the beams (Timoshenko theory) or the plates in bending. To correct this deficiency, so-called transverse shear correction factors are introduced. Finite elements formulated in displacement based on first order theory generally give good results for isotropic and orthotropic structures. They become less precise when applied to composite materials containing several layers with very different anisotropy from one layer to another Topdar et al **[49]**, in which case it would be necessary to impose conditions of continuity on the interfaces. Indeed, transverse shear correction factors, once introduced in the 1st order models in displacement, have solved problems of multilayer structures but their evaluation unfortunately depends on the number of stratifications. To rule out this type of problem forever, high order theories were introduced in the early 1970s. The first theory was proposed in 1969 by Whitney, who assumed a high order displacement field of 3. It Gave precise results but was abandoned because of its theoretical complexity; It requires a large number of Whitney parameters **[51]**. Other theories have appeared later, each of which has advantages and disadvantages, with different formalisms depending on the field of application.

3.2. The classical Laminated Plate Theory (CLPT) of Love-Kirchhoff

It is called a thin plate when the arrow generated by the shear deformations remains negligible in front of the arrow generated by the curvature of the plate. In the case of an isotropic homogeneous plate, the shear rate in the arrow is directly related to the slenderness (L/h). The theory **CLPT** (Classical laminated platform theory) is presents the simplest approach. This theory is based on the hypotheses of Love Kirchhoff [19], according to which a straight line normal to the mean plane of the plate remains perpendicular after deformation **Fig.2**, which amounts to neglecting the transverse shear deformation effects Cugnoni [10].
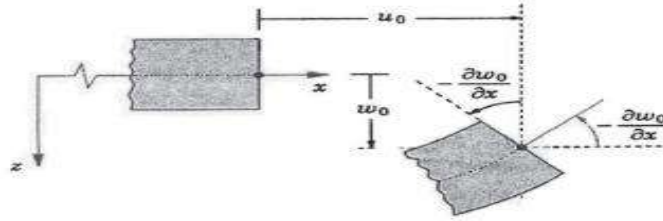

Figure .2. Illustration of the Love-Kirchhoff.

The approximate displacement field used in this formulation is of the following form:

$$u(x, y, z) = u_0(x, y) - z\frac{\partial w_0}{\partial x}, \qquad \textbf{(1a)}$$

$$v(x, y, z) = v_0(x, y) - z\frac{\partial w_0}{\partial y}, \qquad \textbf{(1b)}$$

$$w(x, y, z) = w_0(x, y), \qquad \textbf{(1c)}$$

With u_0, v_0, w_0 : The membrane displacement in the x, y and z directions on the mean plane of the plate (z = 0), w_0: The arrow on the plate, $\dfrac{\partial w_0}{\partial x}$: Rotation due to bending (without shearing).The principal plane of the plate is the plane x, y, and the thickness h is oriented along the axis z = [-h/2, h/2]. Since this model does not account for the transverse shear effect, it gives imprecise results for thick plates.

3.3. The First Order Shear Deformation Thory (FSDT)

First-order shear deformation theory extended the classical plate theory by taking into account the transverse shear effect, in this case the stresses and deformations are constant across the thickness of the plate, which oblige l Introduction of a correction factor. Studies on the first-order shear deformation theory (**FSDT**) can be referred to in (Reissner [37], Mindlin [26]) which led to the Reissner-Mindlin plate model. As well as Timoshenko et al [46], Reddy [38,39].
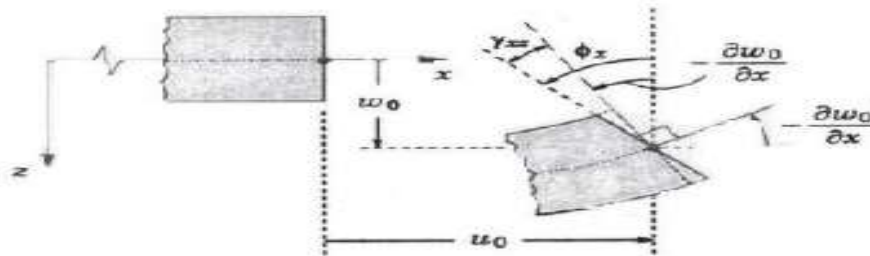

Figure.3. Illustration of the Reissner-Mindlin.

The first-order theory is based on the following displacement field:

$$u(x, y, z) = u_0 + z\phi_x(x, y), \qquad \textbf{(2a)}$$

$$v(x, y, z) = v_0 + z\phi_y(x, y), \qquad \textbf{(2b)}$$

$$w(x, y, z) = w_0(x, y) \qquad \textbf{(2c)}$$

Or u_0, v_0, w_0 are the membrane displacements and (ϕ_x, ϕ_y) the rotations of the normal to the mean plane around the x and y axes, respectively. The displacement field defined in the above expression makes it possible to take up the classical theory of plates described in the last section by replacing, $\phi_x = -\frac{\partial w_0}{\partial x}, \phi_y = -\frac{\partial w_0}{\partial y}$. In order to avoid the introduction of a correction factor, high-order shear deformation theories have been developed.

3.4. The Higher-Order Shear Deformation Theory (HSDT)

This class of finer theories is based on a development of displacement in thickness to order two or more. These theories are particularly well adapted to modeling the behavior of thick plates or short beams, where transverse deformation plays a predominant role. Most of these models use a series development of Taylor Nguyen [35], the high order theory is based on a nonlinear distribution of fields in the thickness. Consequently, the effects of transverse shear deformation and / or normal transverse strain are taken into account. These models do not require correction factors. References on such models can be found in Hildebrand et al [15], Naghdi [33], Reissner et al [39], Reddy [40], Kant et al [23]).
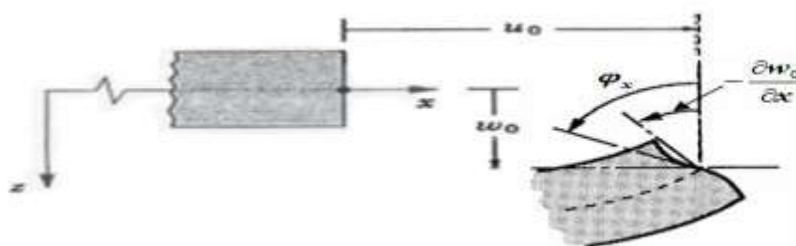

Figure .4. Illustration of the high order plate.

The field of displacement is usually written as follows:

$$u(x, y, z) = u_0 - z\frac{\partial w_0(x, y)}{\partial x} + \psi(z)\varphi_x(x, y), \qquad \textbf{(3a)}$$

$$v(x, y, z) = v_0 - z\frac{\partial w_0(x, y)}{\partial y} + \psi(z)\varphi_y(x, y), \qquad \textbf{(3b)}$$

$$w(x, y, z) = w_0(x, y). \qquad \textbf{(3c)}$$

u_0, v_0, w_0 et (ϕ_x, ϕ_y) are the membrane displacements and the rotations about the **x** and **y** axes, respectively ($\varphi_x = \frac{\partial w_0}{\partial x} + \phi_x$, $\varphi_y = \frac{\partial w_0}{\partial y} + \phi_y$), **Ψ**(z) is a transverse shear function characterizing the corresponding theories. Indeed, the displacements of the classical plate theory (**CLPT**) are obtained by taking **Ψ (z) = 0**, while the first-order theory (**FSDT**) can be obtained by **Ψ (z) = z**.

4. REVIEW OF THE DIFFERENT MODELS OF HIGH-ORDER THEORY

To overcome the limitations of first order theories, several authors propose some important contributions to the development of high order models which have been distinguished in the literature by the expression of the shear function *f(z)*. These models are based on a nonlinear distribution of the displacement fields in the thickness and which allow representing the warping of the cross-section in the deformed configuration **Fig.4**. Whitney [52], Nelson [32], Lo [25] and Touratier [48]. We cite in particular:

- The approach of Ambartsumyan [1]: $f(z) = \frac{z}{2}\left(\frac{h^2}{4} - \frac{z^2}{3h^2}\right)$ **(4)**

- The approach of Reissner [37], Panc et al: $f(z) = \frac{5}{2}z\left(1 - \frac{5z^2}{4h^2}\right)$ **(5)**

- The approach of Levinson, Murthy [27] et Reddy [41,42] :

$$f(z) = z\left(1 - \frac{4z^2}{3h^2}\right) \tag{6}$$

In the Reddy model, the membrane displacement field is cubic and the normal displacement z, is constant Reddy [40]. This model gives a good approximation for the transverse shear stresses with respect to the three-dimensional elastic solution in the homogeneous case Duong et al [14].

The distribution of the transverse shear stresses is parabolic in the thickness. The boundary conditions on the free surfaces are satisfied. Touratier et al [48] proposes the sine model (**SSDT**) which is different from other higher order models since it does not use a polynomial function. A sinusoidal trigonometric function is thus introduced to model the distribution of the shear stresses in the thickness. The transverse shear function can be written as follows:

$$f(z) = \frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right) \tag{7}$$

The transverse shear stresses determined by the models (sine) take a cosine form in the Sinusoidal thickness of the beam. The precision of this model with respect to the exact solution is better than Reddy's theory [40]. Recently, Afaq et al [3] propose an exponential model (**ESDPT)** with a richer kinematics. The transverse shear distribution function is of the following form:

$$f(z) = z.e^{-2(z/h)^2} \text{ et } \varphi_z = 0 \tag{8}$$

The choice of the exponential function allows an even and odd power development of the variable z, whereas the function (sine) Touratier et al [48] allows only an odd power development. The hyperbolic version of the theory of high-order shear deformation (**HSDPT**) developed by Ait Atmane et al. [4] is obtained by taking:

$$f(z) = \frac{\cosh(\pi/2)}{[\cosh(\pi/2)-1]}z - \frac{(h/\pi)\sinh\left(\frac{\pi}{h}z\right)}{[\cosh(\pi/2)-1]} \text{ et } \varphi_z = 0 \tag{9}$$

5. THEORY OF ZIG-ZAG

To better describe the shear deformation of composite materials, some authors have associated the theory of high order with that of ziz-zag Cho et al [5] and Choa et al [6]. The latter is intended precisely to better describe the interface effects. Thus, different models resulting from the layer approach have been proposed. The multilayer is subdivided into substructures (corresponding in fact to each layer or set of layers). We apply to each substructure a first order theory or a high order model. The kinematics of the zig-zag models satisfies a priori the contact conditions and is independent of the number of layers. The main advantage of the displacement field of zig-zag models lies in the good modeling of the distortion of the normal to the deformed surface, as well as in the verification of the continuity conditions, without, however, increasing the number, The order of the fundamental equations of the first order theory. The use of transverse shear correction coefficients is avoided. Based on the concept of Di Sciuva [13], several authors have made significant improvements in the zig-zag model (Murakami et al [28], Averill et al [2], He et al [16], Icardi et al [18], Carrera et al. [9]). The main improvement is the introduction of a nonlinear distribution of displacements. We superimpose the zig-zag field (linear by piece) to a field of displacement of high order (often cubic) **fig.5**. The compatibility conditions are satisfied on the top and bottom surfaces of the plates to reduce the number of parameters [50].
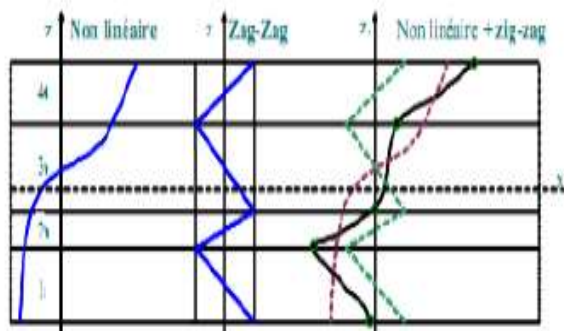
Figure.5. Field of displacement of the high-order zig-zag model [9].

Through our reading of the literature on high order theories, it appears that these are certainly interesting from the point of view of precision, but nevertheless remain costly in terms of computation time and quite complex in terms of formulations.

6. APPLICATION OF HIGH-ORDER THEORY FOR COMPOSITE STRUCTURES AND FGM

6.1. The stratified composite structures

The theories of high order envisaged by certain authors are applicable to certain types of problems (static, dynamic, buckling, ..). The performance of a finite element is related to the theory used. High order theory is distinguished by its polynomial order, its number of coefficients or parameters it generates, and the type of element it uses. Kapania and Raciti [20] provided a detailed synthesis of the shear deformation theories used in statics, vibration and buckling analysis of beams and composite plates. Moita [29] proposed a quadrilateral element at 9 knots and 10 degrees of freedom per nodes. The displacement fields "u" and "v" are cubic with respect to the thickness and "w" is constant. It is particularly effective for calculating the buckling of thick and thin plates. The number of parameters being important, makes the calculation quite cumbersome. Kant et al [22] developed an element based on refined high order theory (especially on sandwich plates). They have defined a displacement field in such a way that "u, v, w" are cubic with respect to the thickness. Each set of two successive layers is divided into a number of sublayers to improve the state of shear stress and to have continuity at the interfaces. Patel et al. [36] treated a shell structure geometrically complicated to the 3rd order, in order to improve the state of deformation. They introduced the zig-zag effect which ensures continuity on the interfaces (the number of parameters increases with the number of layers). Zen Wu et al [54], proposed an interesting theory from the results point of view on constraints. This ensures continuity on interfaces and null conditions (bottom and top). It defines on each layer a different field of motion and uses 11 degrees of freedom per node. It is commonly referred to as "high order shear refined theory". Mechab and Tounsi [30] used high order theories to study the flexural behavior of short beams in laminated composites.

All these theories are very interesting, on the one hand to deal with the problem of discontinuity of the stresses on the interfaces and, on the other hand, to avoid the use of the shear correction factors. The results obtained are generally satisfactory. The only criticism to be made to this type of theory is that they are gourmand in time of calculation Tafla [50].

6.2. The FGM & FGM sandwich structures

With the development of various sectors of technology such as aeronautics, astronautics, national defense industries, nuclear energy, civil engineering and biomedical sectors, gradient materials with functional properties **FGM** are considered as materials Composites the most promising Ichikawa [17].

Wetherhold [53] presented a study based on beam theory to eliminate or control thermal deformations by using **FGM** or **FGM** hybrid materials. Sankar and Tzeng [43] obtained an analytical solution based on elasticity laws for **FGM** beams subjected to transverse loads with an exponential variation in the material properties of the constituents. Subsequently, Sankar [44] developed a theory similar to the Euler Bernouli beam theory for gradient beams of functional property while exponentially varying the elastic properties in order to evaluate the stresses and thermal deformations. Chakraborty et al. [8] have developed a shear-type finite element model to

study the static, free vibration and wave propagation problems for two-material beams fused with **FGM**. In the Zenkour study **[57]**, the static response is presented for a rectangular plate of the simply supported **FGM** type subjected to a uniform transverse load. A generalized theory of high order of shear deformation has been employed which allows to give a correct representation of the transverse shear stress where no correction factor is necessary. This theory is simplified by imposing boundary conditions on the upper and lower surfaces of the plate. The material properties of the plate are varied gradually in the direction of thickness according to a simple distribution based on a power law in terms of volume fractions of the constituents. Chinosi and Croce **[11]**, presented a non-conforming finite element model for the **FGM** rectangular plate bending analysis by Reissner-Mindlin theory **[37,38,39]**. The first objective of this research was to present a general formulation for **FGM** hybrid beams using the generalized theory of shear deformation.

In the Kadoli model **[24]**, the field of motion based on high-order shear deformation theory is applied to study the static behavior of metal-ceramic **FGM** beams under the effect of The Kadoli ambient temperature **[24]**. **FGM** beams with a power variation of the volumetric fraction of the metal or ceramic are considered.

A finite element model of an **FGM** beam was presented to establish the static equilibrium equations using the principle of stationary potential energy Kadoli **[24]**. Numerical results for transverse bending and shear stresses in a moderately thick **FGM**-type beam subjected to a uniformly distributed load for boundary conditions (embedded-embedded) and (simply supported) are discussed. The effects of the variation in the volume fractions of the material properties of the **FGM** (ceramic-metal) beam as a function of the exponent of the power law on the flexural and shear behavior of the beam were also discussed.
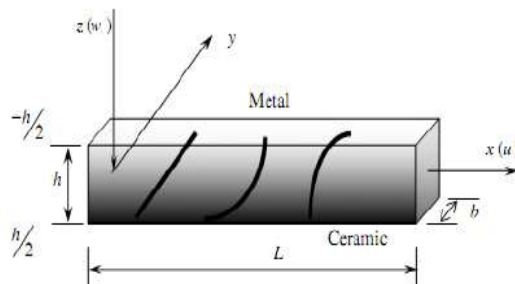

Figure.6. Geometry of the FGM beam with the possibilities of the variation of ceramic and metal through the thickness [24].

The field of displacement based on the high order theory is written as follows:

$$u(x,z) = u_0(x) + z\psi_x - (4/3h^2) * z^3[\psi_x + w_{0,x}], \qquad \textbf{(10a)}$$

$$w(x,y,z) = w_0(x) \qquad \textbf{(10b)}$$

u_0 : Longitudinal displacement of the beam;

w_0 : Vertical transverse displacement;

$\dfrac{dw}{dx}$: Total rotation of section

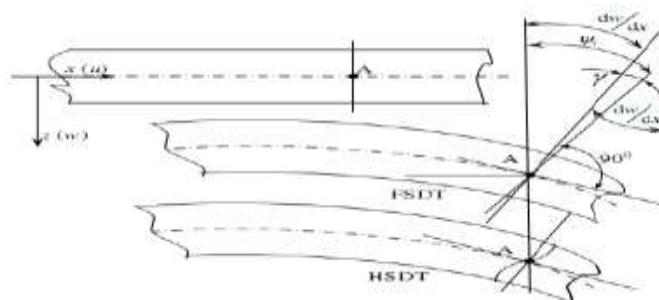ψ_x : Rotation due to transverse deformation

Figure.7. Geometrical description of the deformation of the FGM beam through the thickness.

Recently, Sarfaraz et al **[45]** presented the concept of variational energy by comparing the first order theories with the third-order theory that were used to predict transverse shear deformation across the thickness d An **FGM** plate **Fig.8**. The results are discussed as a function of the thickness and the order "k" which defines a power law considered describing the variation of the material properties of the FGM type plate through the thickness.
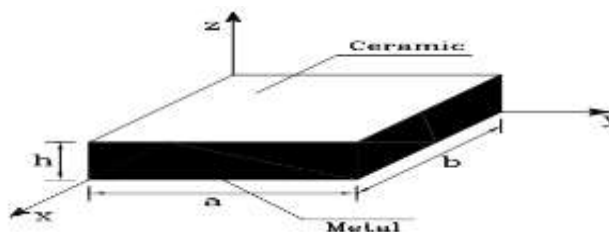

Figure.8. Geometry of a typical FGM plate.

The generalized displacement field is written as follows:

$$u(x, y, z) = u(x, y) + f(z)\frac{\partial w(x, y)}{\partial x} + \psi(z)\phi_x(x, y), \quad \textbf{(11a)}$$

$$v(x, y, z) = v(x, y) + f(z)\frac{\partial w(x, y)}{\partial y} + \psi(z)\phi_y(x, y), \quad \textbf{(11b)}$$

$$w(x, y, z) = w(x, y) \qquad \textbf{(11c)}$$

f(z) **:** is a transverse shear function which can take several forms depending on the theory used.**Ψ***(z)* : is a transverse shear function characterizing the corresponding theories.

(1) Classical theory (CLPT) : $\qquad f(z) = z; \text{ and } \psi(z) = 0$ $\qquad$ **(12)**

(2) First-order theory (FSDT) : $\qquad f(z) = 0; \text{ and } \psi(z) = z$ $\qquad$ **(13)**

(3) Third Order Theory (**TSDT**) : $\qquad f(z) = z^3\left(\frac{4}{3h^2}\right); \text{ and } \psi(z) = z - z^3\left(\frac{4}{3h^2}\right).$ **(14)**

According to Sarfaraz, the results proved that the energy method is able to show that the change in thickness thus of the power "k" which describes the voluminal fraction of the material properties have the effect of modifying the transverse shear behavior of the plate type **FGM** Sarfaraz **[45]**. In 2010, Zenkour presented a sinusoidal theory of shear deformation to study the behavior and thermal resistance of **FGM** sandwich plates Zenkour **[58]**.
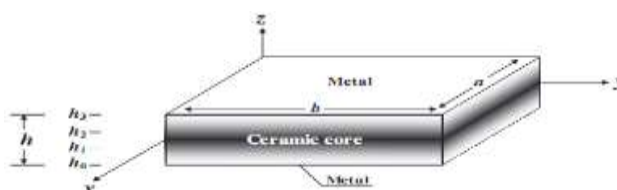

Figure.9. Geometry of FGM sandwich plate.

A comparison of the classical, first order and high order theories was made to describe the shear deformation of the Zenkour plates **[57]**. The study showed that the material properties and the thermal expansions of the symmetrical plates of **FGM** sandwich type vary gradually according to the thickness in terms of the fractions of the volumes of the constituents.

7. CONCLUSION

The classical theory of plates, based on the Kirchhoff hypothesis, makes it possible to describe with precision the fields of stresses and deformations in the thin plates. The validity of the plate theory can be established by comparing the results obtained from this theory with the exact solutions of the elasticity equations. On the other hand, in the case of thick plates (**a/h>10**), classical theory becomes rather poorly adapted to the description of mechanical behavior.

A comparison of the classical, first order and high order theories was made to describe the shear deformation of the plates. The study showed that the material properties of the symmetrical plates of the FGM sandwich type vary gradually according to the thickness in terms of the fractions of the constituent volumes. Using the high order shear deformation theory with a modification in the transverse displacement formulation, the equilibrium equations are derived using the variational approach considering the traction of upper and lower faces of the plates. The mechanical properties of the plate are assumed to vary without interruption in the direction of thickness by a simple distribution of power law in terms of volume fractions of the constituents.

ANNEXE

For the first-order functions, we use that of [Timoshenko 1972], and for the high-order functions we will use different models which are presented, the following table:

Model	Theory	Function
CLPT	Classical	$f(z) = 0$
FSDPT	*First Order*	$f(z) = z$
PSDPT	Parabolic	$f(z) = z\left(1 - \dfrac{4z^2}{3h^2}\right)$
SSDPT	Sinusoidal	$f(z) = \dfrac{h}{\pi} \sin\left(\dfrac{\pi z}{h}\right)$
HSDPT	Hyperbolic	$f(z) = h \sinh\left(\dfrac{p\,z}{h}\right) - z \cosh\left(\dfrac{1}{2}\right)$
ESDPT	Exponential	$f(z) = z.e^{-2(z/h)^2}$
RPT1	High order	$f(z) = z - \dfrac{h}{\pi} \sin\left(\dfrac{\pi z}{h}\right)$
RPT2	High order	$f(z) = z\left[\dfrac{-1}{4} + \dfrac{5}{3}\left(\dfrac{z}{h}\right)^2\right]$

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