Iterative Kalman Filter and Related Algorithms for Non-linear Systems

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ABSTRACT

This paper proposes a comparative analysis of different state estimation techniques on linear and non-linear systems. Estimation is the process of finding a value that is usable even if the subject of interest is uncertain, delayed or corrupted due to noise. An Iterative Kalman Filter has been developed for a class of uncertain discrete-time system with delay. It extends, the KF and EKF to the case in which the underlying system is subjected to norm-bounded uncertainties and constant state delay. The IKF is a robust version of KF, but with the necessary modification to account for the parameter uncertainty as well as delay.

Keywords : Kalman Filter (KF), Extended Kalman Filter (EKF), Iterative Kalman Filter (IKF).

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1. INTRODUCTION

The ultimate goal of algorithms research is to find an optimal solution for a given problem. In this paper we have discussed the performance of Kalman Filter (KF), Extended Kalman Filter (EKF) and Iterative Kalman Filter (IKF) over a system affected by uncertainties and time delay.

2. ALGORITHMS FOR ESTIMATION:

In this section we will discuss in detail the performance of various estimation algorithms along with their strengths and drawbacks.

2.1 The Kalman filter (KF)

The Kalman filter (KF) is a tool that can estimate the variables of a wide range of processes. In mathematical terms we would say that a Kalman filter estimates the states of a linear system. The Kalman filter not only works well in practice, but it is theoretically attractive because it can be shown that of all possible filters, it is the one that minimizes the variance of the estimation error. The KF is an extremely effective and versatile procedure for combining noisy sensor outputs to estimate the state of the system with uncertain dynamics. When applied to a physical system, the observer or filter will be under the influence of two noise sources: (i) Process noise, (ii) Measurement noise.

In order to use a Kalman filter to remove noise from a signal, the process that we are measuring must be able to be described by a linear system.

Suppose we have a linear discrete-time system given as follows:

$$x_{k+1} = Ax_k + Bw_k \qquad (1)$$

$$y_k = Cx_k + v_k \qquad (2)$$

Where $x_k \in \Re^n$ is the system state, $y_k \in \Re^m$ is the measured output, $w_k \in \Re^q$ is the process noise, $v_k \in \Re^p$ is the measurement noise, and A, B and C are known real matrices with appropriate dimensions. Our objective is to design a KF of the from

$$\hat{x}_{k+1} = A_f \hat{x}_k + k_f y_k \qquad (3)$$

Where A_f, K_f are time varying matrices to be determined in order that the estimation error $e_k = x_k - \hat{x}_k$ is guaranteed to be smaller than a certain bound for all uncertainty matrices, i.e., the estimation error dynamics satisfies $E\left((x_k - \hat{x}_k)(x_k - \hat{x}_k)^T \right) \le S_k$, with S_k being an optimize upper bound of filtering error covariance over the class of quadratic filter to be defined later.

$$A_f = A - k_f C \qquad (4)$$

$$K_f = \left(A Q_k C^T \right) R^{-1} \qquad (5)$$

Ultimately we have

$$S_{k+1} = A Q_k A^T - \left(A Q_k C^T \right) R^{-1} \left(A Q_k C^T \right)^T + B W B^T \qquad (6)$$

According to all these result we say that, the filter is a robust quadratic estimator with an upper bound of error covariance S_k.

2.2 Extended Kalman Filter for System with Uncertainties

$$x_{k+1} = \left(A + \Delta A_k \right) x_k + B w_k \qquad (7)$$

$$y = \left(C + \Delta C_k \right) x_k + v_k \qquad (8)$$

Where, $x_k \in \Re^n$ is the system state, $y_k \in \Re^m$ is the observation, $w_k \in \Re^q$ is the noise inherent to the process/system model, $v_k \in \Re^p$ is the noise in the measurement. v_k and w_k are regarded as zero mean, uncorrelated white noise sequence with respective covariances as R_k and Q_k.

$$v_k = N\left(0, R_k \right) \qquad (9)$$

$$w_k = N\left(0, Q_k \right) \qquad (10)$$

The matrix $A_k \in \Re^{nxn}$ and $\Delta A_k \in \Re^{nxn}$ in the difference equation (7) is the dynamics matrix and time-varying uncertainty which relates the state at time step k to the state at time step k+1. The matrix $B \in \Re^{nx1}$ called noise matrix. The matrix $C_k \in \Re^{mxm}$ and $\Delta C_k \in \Re^{mxm}$ in the measurement equation (8) relates the state measurement y. In this chapter a critical issue concerns the uncertainty model used. If the uncertain model used does not give an accurate representation of the true uncertainty in the problem but rather over bounds the true uncertainty then this will lead to an overlay conservative robust filter with a correspondingly poor performance, therefore in order to obtain good results from EKF we are assuming the uncertainty matrix in the following structure.

$$\begin{bmatrix} \Delta A_k \\ \Delta C_k \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} F_k E \qquad (11)$$

Where, $F_k \in \Re^{i \times j}$ is an unknown real time varying matrix and H$_1$, H$_2$ and E are known real constant matrices of appropriate dimensions that specify how the elements of A and C are affected by uncertainty in F$_k$.

Our objective is to design KF in the form of an Equation (12), and determine a gain matrix which minimize the mean square of the error e$_k$,

$$\hat{x}_{k+1} = A_f \hat{x}_k + K_f \left[y_k - \left(C + \Delta C_{ek} \right) \hat{x}_k \right] \quad (12)$$

$$\Delta A_{ek} = \varepsilon_k A S_k E^T \left(I - \varepsilon E S_k E^T \right) E \quad (13)$$

$$\Delta C_{ek} = \varepsilon_k C S_k E^T \left(I - \varepsilon E S_k E^T \right) E \quad (14)$$

$$K_f = \left(A Q_k C^T + \varepsilon_k H_1 H_2^T \right) \left(C Q_k C^T + R_{\varepsilon k} \right)^{-1} \quad (15)$$

$$S_{k+1} = A Q A^T + \varepsilon_k^{-1} H_1 H_1^T + B W B^T - \left(A Q C^T + \varepsilon_k^{-1} H_1 H_2^T \right) \quad (16)$$
$$\times \left(R_{\varepsilon k} + C Q C^T \right)^{-1} \left(A Q C^T + \varepsilon_k^{-1} H_1 H_2^T \right)^T$$

According to all these result we can say that, the filter (12) is a quadratic estimator with an upper bound of error covariance S_k.

2.3 Iterative Kalman Filter for System with Uncertainties and Time Delays:

We use an IKF to estimate the state $x_k \in \Re^n$ of a discrete time uncertain controlled system. The system is described by a linear stochastic difference equation as follows,

$$x_{k+1} = \left(A + \Delta A_k \right) x_k + \left(A_d + \Delta A_k^d \right) x_{k-d} + B w_k \quad (17)$$

$$y = \left(C + \Delta C_k \right) x_k + v_k \quad (18)$$

Where $x_k \in \Re^n$ is the system state, $y_k \in \Re^m$ is the measured output, $w_k \in \Re^q$ is the process noise, $v_k \in \Re^p$ is the measurement noise. In the following v_k and w_k will be regarded as zero mean, uncorrelated white noise sequence with covariance R_k and Q_k.

$$v_k = N \left(0, R_k \right) \quad (19)$$

$$w_k = N \left(0, Q_k \right) \quad (20)$$

The matrix $A_k \in \Re^{nxn}$ and $\Delta A_k \in \Re^{nxn}$ in the difference equation (17) is the dynamics matrix and time-varying uncertainty which relates the state at time step k to the state at time step k+1. The matrix $B \in \Re^{nx1}$ called noise matrix. The matrix $C_k \in \Re^{mxm}$ and $\Delta C_k \in \Re^{mxm}$ in the measurement equation (18) relates the state measurement y. The matrix $A_d \in \Re^{p \times p}$ and ΔA_k^d in the difference equation (17) is the delayed matrix and time varying delayed matrix. In this chapter a critical issue concerns the uncertainty model with time delay used. Earlier we already discussed if the uncertain model used does not give an accurate representation of the true uncertainty in the problem but rather over bounds the true uncertainty then this will lead to an overlay conservative robust filter with a correspondingly poor performance, therefore in order to obtain good results from IKF we are assuming the uncertainty matrix in the following structure.

$$\begin{bmatrix} \Delta A_k \\ \Delta C_k \\ \Delta A_k^d \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} F_k E. \quad (21)$$

Where, $F_k \in \Re^{i \times j}$ is an unknown real time varying matrix and H_1, H_2, H_3 and E are known real constant matrices of appropriate dimensions that specify how the elements of A, A_d and C are affected by uncertainty in F_k.

Our objective is to design KF in the form of an equation (22), and determine a gain matrix which minimize the mean square of the error e_k,

$$\hat{x}_{k+1} = A_f \hat{x}_k + K_f \left[y_k - \left(C + \Delta C_{ek} \right) \hat{x}_k \right] \quad (22)$$

$$\Delta A_{ek} = \varepsilon_k A S_k E^T \left(I - \varepsilon E S_k E^T \right) E \tag{23}$$

$$\Delta C_{ek} = \varepsilon_k C S_k E^T \left(I - \varepsilon E S_k E^T \right) E \tag{24}$$

$$K_f = \left(A Q_k C^T + \varepsilon_k H_1 H_2^T \right) \left(C Q_k C^T + R_{\varepsilon k} \right)^{-1} \tag{25}$$

$$S_{k+1} = (1+\mu_k) \left\{ \begin{bmatrix} A Q A^T + \varepsilon_k^{-1} H_1 H_1^T + B W B^T - \left(A Q C^T + \varepsilon_k^{-1} H_1 H_2^T \right) \\ \times \left(R_{\varepsilon k} + C Q C^T \right)^{-1} \left(A Q C^T + \varepsilon_k^{-1} H_1 H_2^T \right)^T \end{bmatrix} \right\}$$
$$+ \left(1 + \mu_k^{-1} \right) \left\{ A_d M_{k-d}^{-1} A_d^T + \varepsilon_k^{-1} H_3 H_3^T \right\} + B W B^T \tag{26}$$

According to all these result we can say that, the filter (4.22) is a quadratic estimator with an upper bound of error covariance S_k.

3. RESULT

In this section we will consider a simulation example and test the performance of all three algorithms discussed above. Consider the following uncertain discrete time system,

$$x_{k+1} = \left\{ \begin{bmatrix} 0 & -0.5 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \delta \end{bmatrix} \right\} x_k + \begin{bmatrix} -6 \\ 1 \end{bmatrix} w_k$$

$$y_k = \begin{bmatrix} -100 & 10 \end{bmatrix} x_k + v_k$$

Where δ is an uncertain parameter satisfying $|\delta| \le 0.3$. Note that the above system is of the form of system (7)-(8) with,

$$H_1 = \begin{bmatrix} 0 \\ 10 \end{bmatrix}, H_2 = 0, E = \begin{bmatrix} 0 & 0.03 \end{bmatrix}$$

$$P = S = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, W = V = 1$$
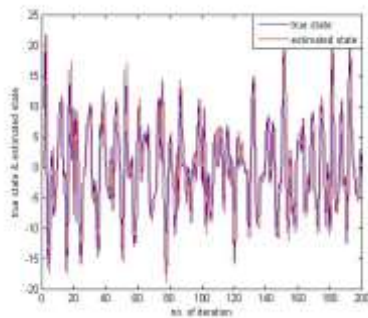

Fig. 1:No. of Iteration vs. True State and Estimated State of KF when uncertainty is zero.
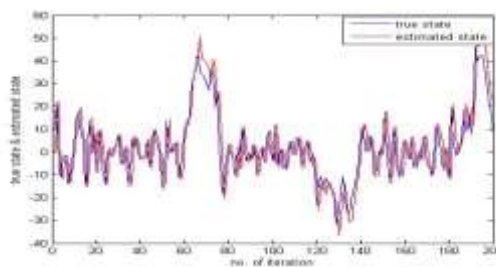

Fig. 2:No. of Iteration vs. True State and Estimated State of KF when uncertainty is 0.3.

Fig. 5.3., shows the graph between no. of iteration vs. true state and estimated state when $\delta = 0$, here the value of estimated state is nearly equal to true state but, when $\delta = 0.3$ the KF give poor result as shown in Fig.2.
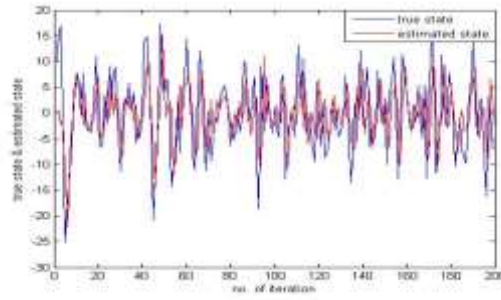
Fig. 3:No. of Iteration vs. True State and Estimated State of EKF when uncertainty is 0.3.

Fig. 3 shows the graph between true state and estimated state with varying time iteration. When δ =0.3, here the value of true state nearly equal to the value of estimated state. From the above figures we can see that estimating the states with EKF is better than KF when system is having uncertainty.
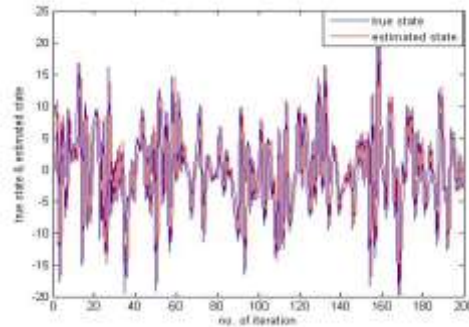

Fig. 4:No. of Iteration vs. True State and Estimated State of KF when delay is zero.
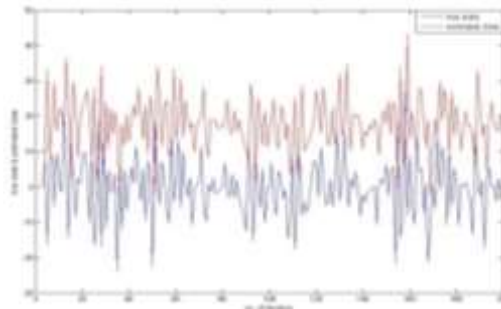

Fig. 5:No. of Iteration vs. True State and Estimated State of KF when delay is 2.

Fig.4 shows that KF performs better when delay is zero but its performance degrades drastically when delay is introduced as shown in fig.5.
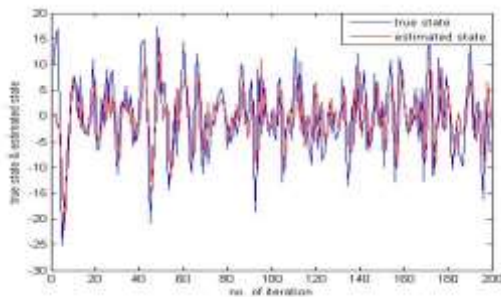

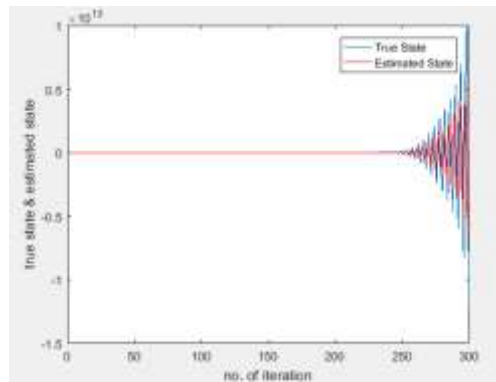**Fig. 6:No. of Iteration vs. True State and Estimated State of EKF when delay is 0.**

Fig. 7:No. of Iteration vs. True State and Estimated State of EKF when delay is 2.
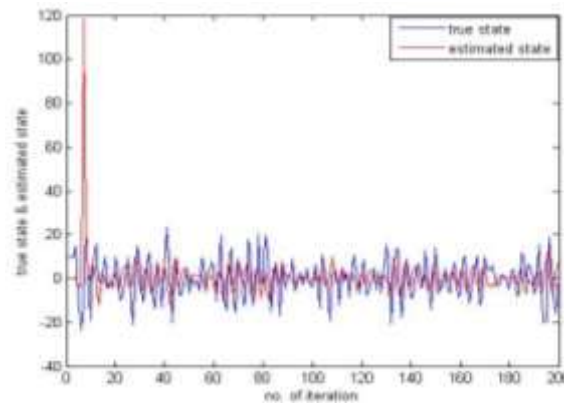

Fig. 8:No. of Iteration vs. True State and Estimated State of IKF when delay is 2 and uncertainty is 0.3.

Fig.6 shows that EKF performs better when delay is zero but it performs poorly when delay is introduced as shown in fig.7.
So we can conclude that EKF performs better in presence of uncertainty but performs poorly when delay is present.
As can be seen from Fig.8, IKF performs much better when delay and uncertainty both are present in the system. So we can conclude that of the three algorithms that we discussed, IKF is the one that performs better for a non linear system.

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