

Comparison of Two Polynomial Geoid Models of GNSS/Leveling Geoid Development for Orthometric Heights in FCT, Abuja

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ABSTRACT

Ellipsoidal heights from GNSS require geoid model for conversion to orthometric height. The geoid model could be global, regional or local. The lack of national geoid model in Nigeria makes development of local geoid very critical to local applications in place of integrated global geoid models. This study compares two polynomial geoid models for terrain representation in the FCT, Abuja. Nine coefficients were used to model the FCT surface for geoid interpolation and orthometric height modeling. Model A involved the use of the 2-D (x, y) positions while model B used 3-D (x, y, Δh) where $\Delta h = (h_{ave} - h_i)$ the difference in average ellipsoidal height (h_{ave}) and each point's ellipsoidal height (h_i). The Δh term is based on the assumption that the geoid varies with topography and may hence possibly lead to some improvements in accuracy of orthometric height determination. DGPS observations were carried out to determine ellipsoid heights. Least squares adjustment was performed to compute the coefficients of the models. Model A achieved standard deviation of $\sigma = 11 \text{ cm}$ while Model B achieved $\sigma = 13 \text{ cm}$. Though, Model B has a term that included highly accurate ellipsoidal height differences (Δh), it has not resulted into any accuracy improvement over the model A. Model A based on 2-D positions is hence the better of the two models. The t-test and hypothesis test at 95% confidence limit, however, showed that the two models did not differ significantly. Model A having lower standard deviation is recommended with GNSS determined ellipsoidal heights to determine orthometric heights within the FCT. This becomes an easy alternative to conventional spirit leveling technique for production of topographical maps, cadastral surveys, and engineering/environmental applications.

Keywords: DGPS, Ellipsoidal Heights, Orthometric Heights, Polynomial Surface, Geoid model, Standard deviation.

1. INTRODUCTION

Heights are defined by their reference surfaces. The basic geodetic surfaces are the earth/topographic surface, the mathematically best-fit ellipsoid approximating the earth surface called the ellipsoid and the geoid which is described as an equipotential surface everywhere perpendicular to direction of gravity. Figure 1 shows the reference surfaces and their relationships. The present height system in Nigeria/FCT is referenced to Mean Sea Level (MSL) which according to Bomford (1980) fails as an equipotential surface because i) its surface is overlain by air, whose pressure varies making the surface not free, ii) the density of water varies, principally with its temperature and salinity among others and concluding that mean sea level is at best only a geoid approximation but departs from geoid by some amounts that are more or less constant over time. Ono (2002) observed that the failure of MSL as a reference surface implies that the Nigerian levelling network cannot be relied on as vertical controls while Fajemirokun (2006) observed that the heights are strictly speaking, not orthometric. Orthometric heights were for centuries obtained by conventional spirit leveling operations but the inherent weaknesses e.g. cost, labour requirements, prone to systematic errors, takes a lot of time over large areas necessitated further search which fortunately was provided by development and application of space technique for Military navigations in point positioning ability. Nwilo (2013) as a result, recommended height modernization in the form of geoid modelling for existing orthometric height in Nigeria.

The GPS uses WGS 84 as datum based on mathematical ellipsoid surface for height; hence we have ellipsoidal height (**h**). The orthometric height (**H**) derived from GPS is a function of the type of geoid model integrated by default to convert the ellipsoidal

height to orthometric height. The geoid model mostly adopted presently is global (EGM2008, EGM96). Global models are designed for global and not for local applications. Odera and Fukuda (2015) opined that global models are too generalized for local applications which points to the need for local geoid development for local applications e.g. geometric geoid for the FCT.

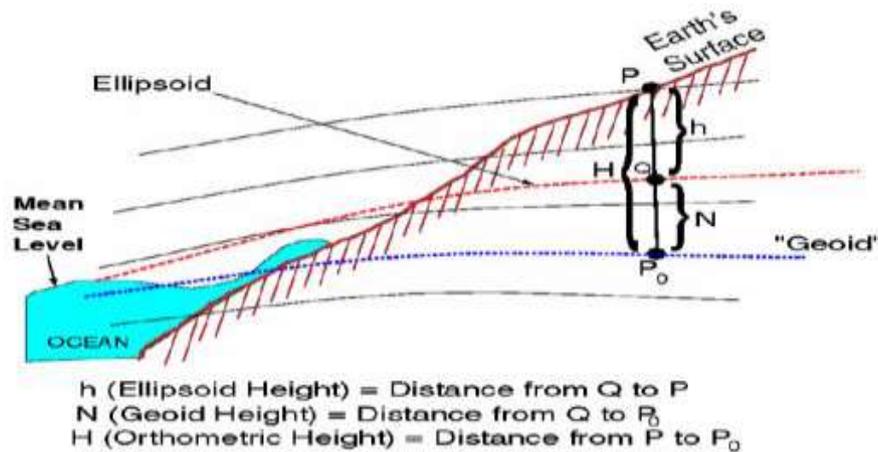


Figure 1: Relationship between the Geoid Height, N, the Ellipsoidal Height, h and the Orthometric Height, H. $N = h - H$. Source: Ono (2009).

The relationship (see Fig 1) between the ellipsoidal height (h) from GNSS observations and H from conventional spirit levelling and the geoid undulation (N) is given by Abdullah (2010), Ono (2009), Uzun and Cakir (2006) and Eteje *et al* (2018) as:

$$N = h - H \tag{1}$$

and

$$H = h - N \tag{2}$$

Equation (2) is used to transform ellipsoidal height to orthometric height. It should be noted however, that the ellipsoid is a known mathematical surface while the geoid surface is the surface of reference being developed from geoid modelling. Ezeigbo (2006) observed that absence of national geoid model puts a limitation to realizing the full benefits of GNSS in Nigeria. Ezeigbo (1990) investigated gravimetric geoid model for Nigeria and achieved 1m accuracy which is inadequate for local applications. Uzodinma *et al.* (2014) also in a study using EGM2008 along with levelled orthometric heights arrived at accuracy of about 1.019m. Epuh *et al.* (2016) reported a 2.2m difference using GPS and levelling in Gongola Basin area. These values from global models are certainly not adequate for local applications and hence the need to develop geometric geoid model for GPS user community. This study, therefore, compared two polynomial surfaces for geometric geoid modelling of FCT in place of global model. One of the models included an ellipsoidal height difference term (Δh) as observed in Okiwelu *et al.* (2011) that geoid varies generally with topography.

Generally, Kirici and Sisman (2017) observed that polynomials can be represented as follows:

$$N_{x,y} = \sum_{i=0}^m \sum_{j=k-1}^n a_{i,j} x^i y^j \tag{3}$$

where $a_{i,j}$ polynomial coefficient
 m degree of polynomial
 (x, y) plane coordinates

The model A is a function of the 2-D positions i.e. easting (x) and northing (y) of points used for data acquisitions while model B used 3-D easting, northing and ellipsoidal height differences between mean ellipsoidal height (h_{ave}) and ellipsoidal height (h) of each point. (x, y, Δh) and are shown respectively as:

$$\text{Model A, } N = a_0 + a_1x + a_2y + a_3x^2 + a_4y^2 + a_5xy + a_6x^2y + a_7xy^2 + a_8x^2y^2 \tag{4}$$

$$\text{Model B, } N = a_0 + a_1x + a_2y + a_3x^2 + a_4y^2 + a_5xy + a_6x^2y + a_7xy^2 + a_8\Delta h \tag{5}$$

$$\Delta h = h_{ave} - h \tag{6}$$

1.1 Aim and Objectives of study

The aim is to evaluate the two geometric geoid models for orthometric height in FCT with a view to recommending which model to adopt by the GPS user community for various applications. The objectives were: to acquire ellipsoidal height (h) of controls using DGPS observations; to determine geoidal undulation N and develop Microsoft excel program for interpolation of N and hence obtain orthometric height; to compare the orthometric heights from the two models by using t-test statistics.

1.2 Study Area

The study area is the Federal Capital Territory (FCT), Abuja, Nigeria. The FCT (Fig. 3) lies between latitude 8° 15'N to 9° 12' N and longitude 6° 27'E to 7° 23'E and located in central region of Nigeria (Fig. 2). The twenty-four multi-network controls selected for observations are all located within the FCT.

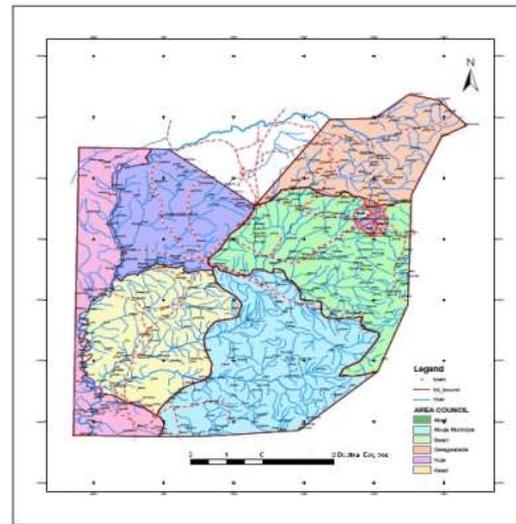


Figure 2: Map of Nigerian States and FCT Figure 3: Map of FCT and Six Area Councils
Source: Arcinfo Shapefile 2010 (ESRI)

2. METHODOLOGY

The dual frequency DGPS Hi-Target V30 Pro receiver with accessories was selected for field measurements. Reconnaissance was done to confirm physical status/existence of controls selected during office planning including access to their locations. The DGPS was used in static mode (2 hours) per station with five seconds epoch rate to acquire data for ellipsoidal coordinates of the selected controls.

2.1 Data Processing

Static observations were post-processed using MagicGNSS, CSRS-PPP and OPUS online software. The average ellipsoidal height was computed and used for geometric geoid development. Table 1 shows the results of computed average ellipsoidal height.

Table 1: Average Ellipsoidal Heights and Computed Geoid Undulation.

CONTROL POINTS	COORDINATE REGISTER VALUE			post processing	Undulation (N)
	EASTINGS (m) x	NORTHINGS (m) y	ORTHO HEIGHTS, H (m)	AVERAGE h (m)	N=h-H (m)
FCC11S	331888.114	998442.043	485.447	509.396	23.949
FCT260P	255881.175	993666.807	201.944	224.74	22.787
FCT103P	340639.766	998375.578	532.558	556.836	24.278
FCT12P	333743.992	1008308.730	735.707	760.192	24.485
FCT19P	337452.408	996344.691	635.644	659.824	24.18
FCT2107S	308926.908	989748.256	316.092	342.103	26.041

FCT2168S	310554.927	1009739.930	431.087	455.274	24.187
FCT24P	322719.776	1001884.850	453.804	477.987	24.183
FCT276P	351983.716	1025998.314	625.572	649.848	24.276
FCT4154S	329953.882	1003831.280	476.981	501.232	24.251
FCT4159S	326124.422	1003742.860	452.230	476.553	24.323
FCT66P	299148.035	998114.283	297.111	321.115	24.004
FCT9P	329821.512	1007612.091	497.253	521.693	24.440
FCT35P	322183.380	992926.363	427.171	451.299	24.128
FCT57P	303234.270	992916.402	323.844	347.795	23.951
FCT4028S	330164.634	1001388.240	449.592	473.942	24.35
FCT53P	308943.361	993406.773	351.943	375.955	24.012
FCT4652S	329441.767	997474.808	462.711	487.113	24.402
FCT162P	270791.291	934625.533	189.696	215.091	25.395
FCT130P	330982.584	952889.869	695.608	719.383	23.775
FCT2327S	282526.612	973821.470	183.287	207.482	24.195
FCT2652S	271370.273	945385.429	138.952	163.741	24.789
FCT2656S	272644.591	941062.460	204.724	229.229	24.505
FCT83P	332954.205	987231.606	568.752	592.819	24.067
XP382	284074.729	983364.863	274.586	298.390	23.804

2.2 Mathematical Model

A mathematical model is a set of one or more equations that properly represents reality e.g. a polynomial equation to represent a geoid surface for modelling of geoid undulation (N) and by implication orthometric heights. Observation equation was written for each observation in the form given by Ono (2002) as:

$$V=AX+L \tag{7}$$

where A is a design matrix; V is residual
 X is the vector of unknown parameters/coefficients
 L is measurements of geoid undulations (N=h- H).

2.3 Least Squares Principles

For redundant observations in survey measurements, the least squares principles based on minimization of sum squares of weighted residuals is generally represented by Ono (2002) as:

$$\sum wv^2=w_1v_1^2+w_2v_2^2+w_3v_3^2+\dots+w_nv_n^2 \text{ _____ } \min \tag{8}$$

where w_i is the weight.

The solution of the least squares formulation is given by

$$X=(A^TWA)^{-1}(A^TWL) \tag{9}$$

$$X = (A^T A)^{-1} (A^T L) \tag{9a}$$

(9a) is for unit weight due to equal reliability of observations.

Standard deviation of observations (σ) is given as:

$$\sigma = \sqrt{\frac{v^2}{n-1}} \tag{10}$$

The constants $a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7$ and a_8 for the models were determined with least squares method using online matrix calculator (Huobi.pro). The values of the constants are given below as:

For **Models A and B**, i.e. equations (4) and (5) respectively, we have:

$$X = \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \end{pmatrix} = \begin{pmatrix} 24.224890121000000000 \\ -0.00002409340580587179 \\ -0.00008013607770038382 \\ 0.00000000009699046795 \\ 0.00000000370280953876 \\ 0.0000000116702184889 \\ -0.0000000000021600943 \\ -0.0000000000045716237 \\ 0.0000000000000000886 \end{pmatrix} \quad X = \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \end{pmatrix} = \begin{pmatrix} 23.56212134646793537362 \\ 0.0000319500274725242 \\ 0.0000922396948498062 \\ -0.00000000007402329853 \\ -0.0000000035342910793 \\ -0.0000000024412997894 \\ 0.0000000000000768716 \\ 0.00000000000008788422 \\ -0.0003574116805908969 \end{pmatrix}$$

2.4 Geometric Geoid Development

Microsoft excel 2010 was used to program the two polynomial surface models for interpolation of geoid undulation (N) and orthometric height (H). Ziggah, *et al.* (2013) was used for centroid computation in the geometric geoid program development for geoid interpolation and orthometric height computation in excel spreadsheet. The results are shown in Table 2: Existing, model A and model B orthometric heights.

Table 2: Orthometric Heights for Existing, Models A and B

CONTROL POINTS	EASTINGS (x) m	NORTHINGS (y) m	ORTHO HEIGHTS H(m) Existing	MODEL A H (m)	MODEL B H (m)
FCC11S	331888.114	998442.043	485.447	485.161	485.155
FCT260P	255881.175	993666.807	201.944	201.963	201.947
FCT103P	340639.766	998375.578	532.558	532.681	532.711
FCT12P	333743.992	1008308.730	735.707	735.826	735.808
FCT19P	337452.408	996344.691	635.644	635.703	635.644
FCT2168S	310554.927	1009739.930	431.087	431.087	431.101
FCT24P	322719.776	1001884.850	453.804	453.807	453.666
FCT276P	351983.716	1025998.314	625.572	625.580	625.425
FCT4154S	329953.882	1003831.280	476.981	476.896	476.906
FCT4159S	326124.422	1003742.860	452.23	452.269	452.219
FCT66P	299148.035	998114.283	297.111	296.925	296.921
FCT9P	329821.512	1007612.091	497.253	497.334	497.366
FCT35P	322183.38	992926.363	427.171	427.252	427.277
FCT57P	303234.270	992916.402	323.844	323.747	323.807
FCT4028S	330164.634	1001388.240	449.592	449.642	449.649
FCT53P	308943.361	993406.773	351.943	351.944	352.009
FCT4652S	329441.767	997474.808	462.711	462.916	462.886
FCT162P	270791.291	934625.533	189.696	189.694	189.809
FCT130P	330982.584	952889.869	695.608	695.579	695.622
FCT2327S	282526.612	973821.470	183.287	183.221	183.457
FCT2652S	271370.273	945385.429	138.952	138.960	139.123
FCT2656S	272644.591	941062.460	204.724	204.715	204.484
FCT83P	332954.205	987231.606	568.752	568.91	568.778
XP382	284074.729	983364.863	274.586	274.399	274.441

Standard deviation (σ) is a key accuracy indicator and for model A, $\sigma = 11\text{cm}$ while model B has $\sigma = 13\text{cm}$. This implies that both models are of comparable accuracies and can be used interchangeably for determination of orthometric heights in the study area by GNSS users.

This study has also indicated that the geoid undulation can also be obtained as a function of either 2-D (x, y) or 3-D (x, y, Δh) positions.

The standard deviation values computed and compared within the permissible limits given by American Society of Photogrammetry and Remote Sensing (ASPRS 1993) specifications as shown in Table 3 for topographic elevation accuracy requirement.

Table 3: ASPRS Topographic Elevation Accuracy Requirement for Well-Defined Points

Contour Interval (M)	Class I (M) High Accuracy/Standard Deviation Accuracy	Class II (M) Standard Deviation	Class III (M) Standard Deviation
0.5	0.08	0.16	0.25
1.0	0.17	0.33	0.5
2.0	0.33	0.67	1.0
4.0	0.67	1.33	2.0
5.0	0.83	1.67	2.5

Source: American Society of Photogrammetry and Remote Sensing (ASPRS 1993)

From Table 3, it is seen that both models can be used to produce topographical plan of 1 m contour interval for base maps, survey plans for engineering and environmental applications. For less accurate survey and engineering requirements, the models may even be used to produce maps at 0.5m contour intervals e.g. for road construction works, cadastral surveys, preparation of master plan or land use classification maps.

2.5 Coefficient of correlation (R) and coefficient of Determination (R²)

R was computed for the determination of fit of model to the FCT surface while R² indicates the percentage of variation explained by the polynomial model. Edan *et al.* (2014) observed that the coefficient of determination should be within the range of 0 < R² < 1. The closer R² is to 1, the better the fit to the observations measurements.

2.6 Correlation Coefficient (R) between orthometric heights (based on MSL and Model B based on geoid)

Adamu and Johnson (1974) and several standard texts used (11) for computing R:

$$R = \frac{n(\sum x) - (\sum x)(\sum y)}{\sqrt{((n \sum x^2 - (\sum x)^2) - (n \sum y^2 - (\sum y)^2))}} \tag{11}$$

where

$$x = H_{modelB} \quad y = H_{MSL}; \quad n = \text{no of stations (24 in this study)}$$

R = correlation coefficient is used to estimate quality of fit of the MSL and Model B based orthometric heights.

From Table 2, the correlation coefficient R was computed from the above relationship and the computed coefficient of correlation R = 1 which implies very strong possible agreement while coefficient of determination R² = 1 (100%) which is an indication of how well the models explain and predict the geoid undulation and hence the orthometric height. The unadjusted R² is used to identify which predictors should be included in the model or discarded. In this study unadjusted R² = 1, therefore all the predictors (x, y, Δh) are retained in the models.

2.7 Computations

This involved computation of mean, standard deviation (S_A) and pooled estimates (S_{AB}) from the following relationships:

$$\text{Mean } \bar{y} = \sum y_i / n \tag{12}$$

$$S_A = \sqrt{((\bar{y} - y)^2 / (n_A - 1))} \tag{13}$$

$$S_{AB} = \sqrt{((n_A - 1)S_A^2 + (n_B - 1)S_B^2) / (n_A + n_B - 2)} \tag{14}$$

Statistical t-test is used for comparison of two things/data sets and can be calculated from

$$\text{Calculated } t = \frac{|\bar{y}_A - \bar{y}_B|}{S_{AB} \sqrt{\left(\frac{1}{n_A} + \frac{1}{n_B}\right)}} \tag{15}$$

From Table 2 showing the orthometric heights from the two models, we have $t_{cal} = 0$ and from t table at degrees of freedom=46 and 95% critical/confidence level, $t_{table} = 2.013$.

2.8 Hypothesis testing

The null hypothesis H_0 is given by

H_0 : The mean H of model A is equal to the mean H of model B

H_1 : The mean H of model A is not equal to the mean H of model B

Decision rule is given as: if $t_{cal} > t_{table}$, reject H_0 and accept H_1

Since $t_{cal} < t_{table}$ i.e. 0 is less than 2.013, we accept H_0 to imply that there is no difference between the mean orthometric heights of the two models.

2.9 Products from Models A and B

Products from the two models are:

i) Contour maps

The orthometric heights from both models, A and B are shown in Fig. 4, Fig. 5 and Fig. 6. Using surfer 8 software and kriging interpolation, contours are generated for both models and are shown in Figures.

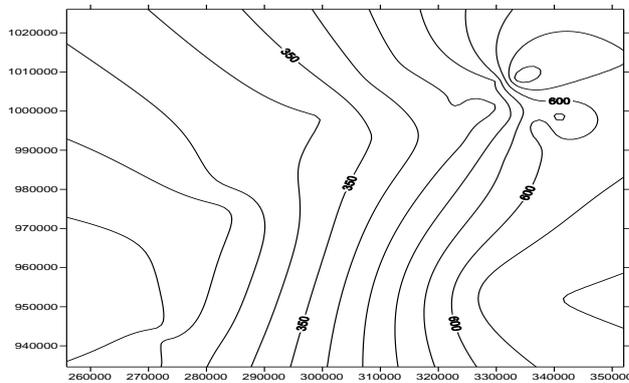


Fig. 4: Contour Map of Existing Orthometric Heights

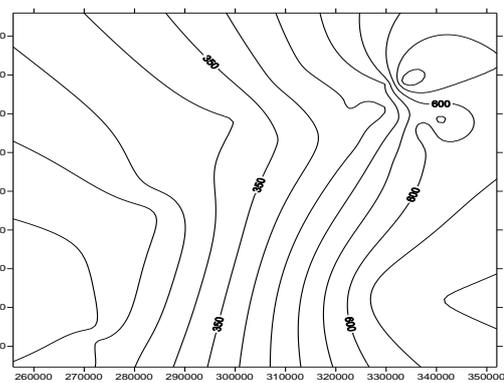


Fig. 5: Contour Map of Model A Orthometric Heights

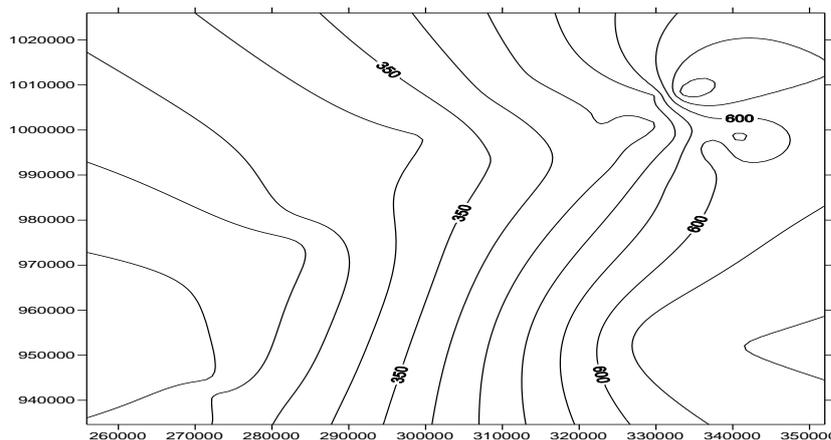


Fig. 5.6: Contour Map of Model B Orthometric Heights

ii) Digital Elevation Models (DEM)

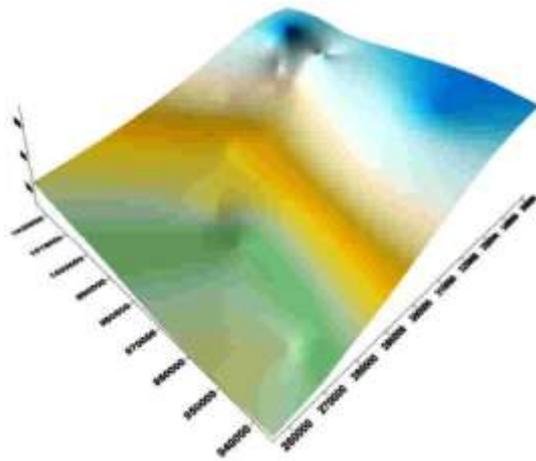


Fig.7: Existing Orthometric Height DEM

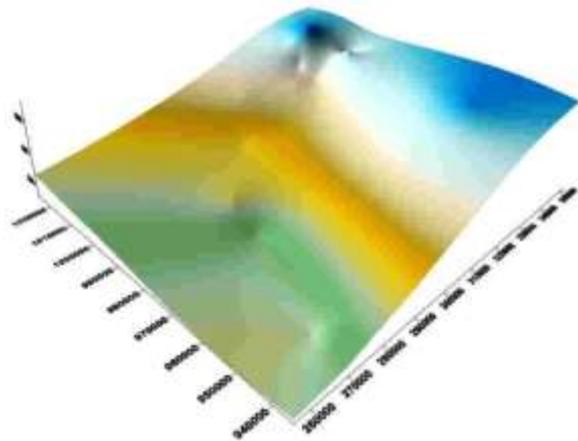


Fig.8: Orthometric Height of MODEL A DEM

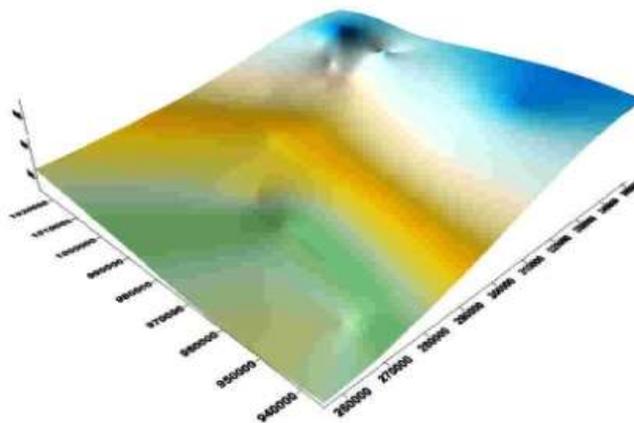


Fig.9: Orthometric Height of MODEL B DEM

3. ANALYSIS OF RESULTS

The standard deviation for model A is 11cm while that for model B is 13cm. This simply means model A based on 2-D position is better for orthometric height determination using DGPS relative technique than model B that is based on 3-D positions. The standard deviation of B has not led to improvement of accuracy over model A indicating that though the geoid is assumed to vary with terrain, it may not necessarily lead to improved accuracy over model A despite the fact that each model has $R^2 = 1$ for acceptable predictive ability/capacity.

The t-test computed and compared with t-critical values for comparison of the two models and hypothesis test also showed acceptance of the null hypothesis H_0 to imply that there is no significant difference between the means of the models. This may be interpreted as confirmation that geoid varies with the topography and imply that the 2-D coordinates (x, y) is adequate for polynomial development of geometric geoid model within the study area.

4. CONCLUSIONS

From the results of this study, Polynomial model was adopted for orthometric height modelling in FCT with model A recommended for cadastral, engineering/environmental, planning and mapping applications that do not require high precisions e.g. in micro-geodetic studies. The ellipsoidal height combined (h) with the existing orthometric height (H) collected from Surveying and Mapping Department of FCDA was used to compute the geoid undulation (N) of each point. Model B that included a difference of ellipsoidal height (Δh) term did not improve the accuracy of orthometric height determination when compared with model A.

Developed model A with DGPS ellipsoidal height will serve as replacement/alternative for conventional third order levelling for orthometric height determination in geospatial data acquisition in engineering and large scale mapping applications instead of reliance on the global models.

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